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ABSTRACT

The Mathematics 16 program provides for the development of essential concepts, skills, and attitudes required for effective computation and problem solving. The program is activity-based, and addresses the need for students to be able to transfer and apply specific mathematical concepts and skills to more generalized situations in everyday life and the world of work. A focus on the use of technology throughout the program assists students to develop the ability to use calculators and computers in performing routine tasks more easily replicated by these technologies. This guide contains the program of studies and curriculum guide for the Science 16 program. Contents include chapters which are entitled: (1) "Rationale"; (2) "Philosophy"; (3) "Goals for the Mathematics 16 Program"; (4) "Model for the Mathematics 16 Program"; (5) "Interpersonal Skills and the Social Sphere"; (6) "Required and Elective Components"; (7) "Planning"; (8) "Learning Resources for Mathematics 16"; (9) "Methodology"; (10) "Evaluation"; (11) "Scope and Sequence"; (12) "Program of Studies/Presentation of Content"; and (13) "Suggested Options." (KR)

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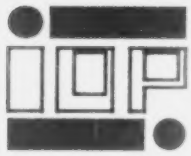

INTEGRATED OCCUPATIONAL PROGRAM

Program of Studies/Curriculum Guide

ED325306

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MATHEMATICS 16

INTERIM – 1990

Curriculum

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ADDENDUM

Addendum to Integrated Occupational Program
Program of Studies/Curriculum Guides 16-Level Courses

COMMUNITY PARTNERSHIPS

School administrators and teachers are advised that the **APPLICATION FOR APPROVAL OF WORK SITES/STATIONS** (Parts A and B) must be completed for each work site/station at which an Integrated Occupational Program (I.O.P.) student is placed.

In addition, each student involved in any off-campus I.O.P. learning activity must be registered by the school/school jurisdiction as being placed in an approved work site/station.

Specific procedures and legislation about work experience (community partnerships) is provided in the following documents:

Alberta Education Program Policy Manual
(Available upon written request from Central Support Services,
11160 Jasper Avenue, Edmonton, Alberta, T5K 0L2.)

Work Experience Program: Teacher Resource Manual Junior and Senior High School,
Alberta Education, 1990
(Available from the Learning Resources Distributing Centre,
12360-142 Street, Edmonton, Alberta, T5L 4X9.)

Mathematics 16

Program of Studies/Curriculum Guide

Grade 10

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NOTE

CURRICULAR DOCUMENT FORMAT

To provide educators with a comprehensive overview of the Integrated Occupational Program, all I.O.P. curricular documents have adopted the format of combining the Program of Studies and Curriculum Guide into one document. The shaded statements or segments within this document indicate the prescriptive contents of the Program of Studies. All other advice and direction provided are suggested only.

The terminology and format used in this document reflects policy in effect when I.O.P. curriculum development began in 1987.

METRICATION POLICY

It is the policy of Alberta Education that "SI units become the principal system of measurement in the curriculum of the schools in the province". In preparing students for transition to the workplace where imperial/U.S. measurements may still be in use, both SI metric and other units of measurement are addressed in the practical arts/occupational component of the Integrated Occupational Program.

The comparison/teaching of metric units with other units of measurement should be restricted to those that are relevant to student needs as reflected by common usage in course-related workplaces.

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RATIONALE

The Integrated Occupational Program (I.O.P.), developed as an outcome of the *Secondary Education in Alberta* (June 1985) policy statement, is a program for students who have experienced difficulty in learning. The program consists of both core and complementary courses designed to develop concepts, skills and attitudes necessary for everyday living.

The Integrated Occupational Mathematics Program provides a two-course sequence in mathematics known as Mathematics 16-26. Mathematics 16 allows students to meet the credit requirements of the Certificate of Achievement. Mathematics 26 allows students to develop their knowledge and skills in mathematics more fully, and facilitates transfer to the General High School Diploma Program.

The Mathematics 16-26 Program provides for the development of essential concepts, skills and attitudes required for effective computation and problem solving. The program is activity-based, and addresses the need for students to be able to transfer and apply specific mathematical concepts and skills to more generalized situations in everyday life and the world of work. The program is intended to foster an appreciation of mathematics for its usefulness and relevance, and thus motivate students to participate in the learning process.

Students need to learn to cope with the rapid pace at which change occurs in today's home and work environments. An emphasis on effective strategies for problem solving will enable students to develop systematic approaches for dealing with new and unfamiliar situations. Students also need to understand how calculators and computer technologies are used in everyday situations involving computation and problem solving. A focus on the use of technology throughout the program will assist students to develop the ability to use calculators and computers in performing routine tasks more easily replicated by these technologies. The demands of daily living require that computation be performed by a variety of methods (i.e., mental arithmetic, paper and pencil, the calculator, estimation). The emphasis placed on computational facility and estimation will help students to select and use computational procedures that are appropriate in a variety of practical situations.

Students are often unaware of strategies they may generate and employ to become more efficient in their cognitive functioning. Evidence supports, however, that students with learning difficulties can perform strategically if taught to do so. Thinking strategies that are emphasized throughout the program will foster effective behaviours in planning, organizing and self-monitoring. As students learn to understand and control the outcome of tasks demanded of them, confidence in taking risks and accepting challenges will further their ability to solve problems and make responsible decisions in everyday life.

PHILOSOPHY

The Integrated Occupational Mathematics Program focuses first and foremost on the needs of the learner. As attitude and self-esteem are powerful influences over learning, the program must foster in each student a positive self-concept and a positive attitude toward learning. The concepts, skills and strategies delivered by the program must:

- provide meaningful and relevant experiences
- be appropriate to student ability
- provide for student success.

Students vary in the way they receive, process, recall, apply, and communicate information. Each student has a preferred way to approach learning tasks. Instructional planning and delivery must include careful assessment of each student's developmental characteristics, knowledge, skill, and preferred way of learning. Adjustments to instructional delivery may often be necessary to ensure that individual student needs are being met.

An integrated approach suggests the linking together of various mathematical skills and strategies into meaningful activities and applications. Abstract concepts and ideas will take on new meaning and significance to students when applied to daily experiences. Organization of mathematics instruction into "themes" is intended to advance the notion of "holistic" learning, relative to both mathematics and the student. Discrete skill instruction is appropriate when specific deficiencies are noted.

Although students are at various stages of cognitive development, most will continue to use concrete operational thinking. Students will depend on personal experience and personalized content to link new ideas with prior knowledge. As the process of analysis must be based on tangible experience, learning activities should begin at the concrete level. High emphasis should be placed on experiential learning involving manipulatives and hands-on activities. Specific skills and concepts should be developed after establishing a need for their use through learning activities involving three levels of instructional technique:

- concrete (e.g., use of models)
- transitional (e.g., pictorial representation)
- formal (e.g., symbolic representation).

Direct assistance must be provided to the learner in progressing from the concrete level of thinking to the more abstract thought processes. Appropriate strategies for providing this assistance have been included in this *Program of Studies/Curriculum Guide* and the corresponding *Teacher Resource Manual*.

The mathematics program must address the realities of a technological age in developing the concepts, skills and attitudes that students will use in everyday life and in the world of work. Current and future demands of our rapidly changing society suggest that the program place increased emphasis on the:

- development of number sense and computational facility
- use of technologies such as the calculator and computer
- application of what is learned to a variety of situations within a changing society.

These learning goals suggest a broader context for instruction in mathematics, and provide a focus for learning activities that are suggested throughout the program.

GOALS OF THE MATHEMATICS 16 PROGRAM

The Integrated Occupational Mathematics Program is designed to assist students in developing and maintaining:

- positive and realistic self-images
- constructive relationships with others
- positive attitudes toward mathematics and lifelong learning.

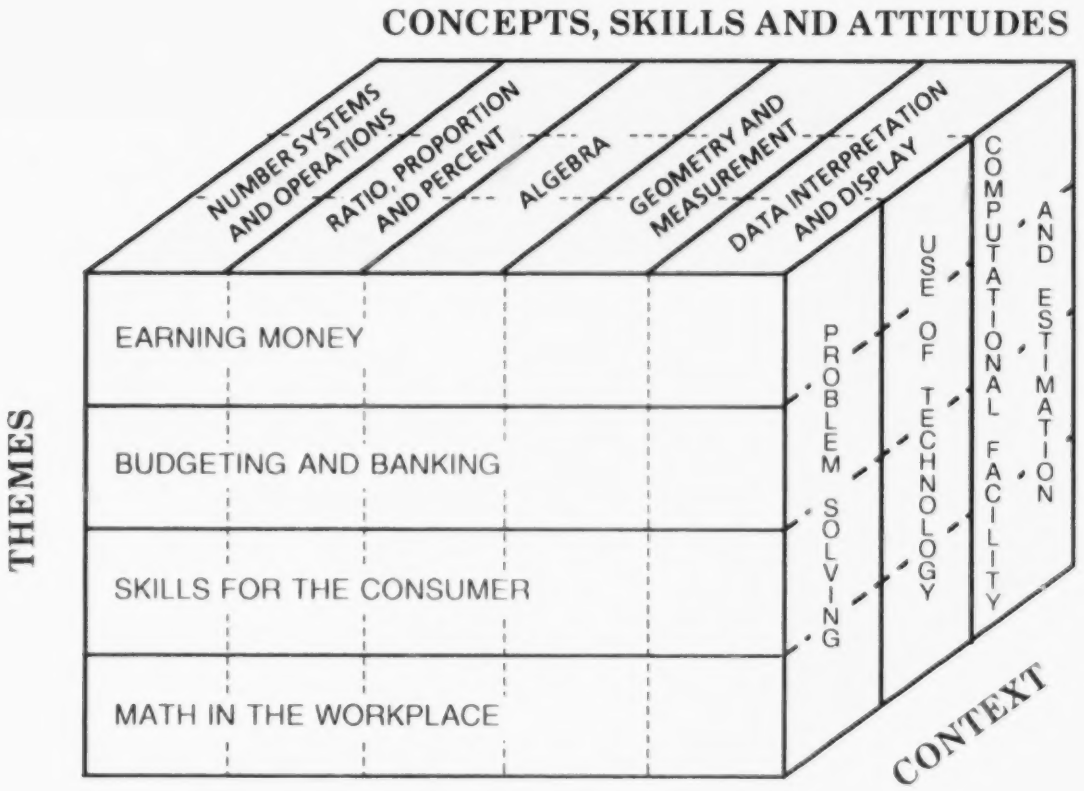
Within the Mathematics 16 Program, students will be expected to:

- develop essential concepts, skills and attitudes of mathematics that are required for responsible participation in the home, the school, the community and the workplace
- apply mathematical concepts and skills to daily life and occupational situations that are experienced both inside and outside the mathematics classroom
- develop critical and creative thinking skills, and apply these skills through a problem-solving process to a variety of practical situations
- develop the ability to use technology in its various forms
- develop communication skills that are used when learning mathematics and solving practical problems.

MODEL FOR THE MATHEMATICS 16 PROGRAM

The model for the Mathematics 16 Program illustrates an integration of program dimensions, thus emphasizing a "holistic" approach to instruction and learning. Three dimensions that provide a basis for program planning are represented on the cube.

- Concepts, Skills and Attitudes
- Context for Instruction
- Themes



The themes have been deliberately placed on the "face" of the cube to highlight their importance in planning for relevant and meaningful learning experiences.

CONCEPTS, SKILLS AND ATTITUDES

Concepts, skills and attitudes reflect the content of Mathematics 16, and provide a structure that will enable students to attain desired mathematical learnings. Five major content strands (i.e., number systems and operations, ratio/proportion and percent, algebra, geometry and measurement, data interpretation and display) have been established on the basis of frequent task demands placed upon students in everyday life. These strands represent a consolidation and extension of concepts and skills included in the Integrated Occupational Mathematics Programs for Grades 8 and 9.

This dimension of the program highlights the developmental nature of mathematical competencies, and promotes a diagnostic approach through the explicit identification and sequencing of prescribed skills within each strand. Skill deficits can be readily determined and activities planned, which will enable individual students to progress from their current level of operation to the next level of functioning.

Concepts, skills and attitudes are outlined in further detail within the "Program of Studies/Presentation of Content" section of this document.

CONTEXT FOR INSTRUCTION

The dimension of the model highlighting context provides for an element in learning that transcends and permeates all that is done within the mathematics program. The context for instruction will foster the development of knowledge and skills that enable students to interpret and process information in their environment relative to the demands of everyday life.

It is intended that all themes foster the development of strategic behaviours as they relate to problem solving, use of technology, computational facility and estimation. Research strongly supports the teaching practice of modelling appropriate strategies, discussing those strategies the students presently use, and encouraging students to develop additional strategies. These techniques are beneficial as students may not spontaneously generate their own learning strategies, and may remember and use teacher taught strategies more often than those acquired incidentally.

The context for instruction is outlined in further detail within the Methodology section and the Program of Studies/Presentation of Content (see "Problem Solving", "Use of Technology", "Computational Facility and Estimation").

THEMES

Themes provide the setting in which discrete concepts, skills and attitudes are linked together into meaningful activities. These activities focus attention on a particular topic or concern, and provide students with practical experiences that contribute to responsible participation at home, at school, in the workplace, and in the community.

Using themes facilitates the acquisition of skills in a way quite different from the "skills-based" approach favoured by many conventional curricula. For example, a skills-based unit on "Decimals" may focus instruction entirely on a particular set of skills within one strand (e.g., operations with decimals). In contrast, a thematic unit on "Skills for the Consumer" will include the use of skills from a variety of strands (e.g., decimals, measurement, ratio).

The thematic approach provides for the natural integration of a variety of skills, and allows the student to solve problems and make decisions that relate to real issues affecting their lives. A well-constructed theme also allows for activities in all levels of thinking (i.e., recall, application, analysis, synthesis, and evaluation). Other advantages of thematic planning and organization of content include:

- provision for cumulative development of background knowledge and skill, enabling students to relate and transfer learning from one day to the next
- opportunity to remediate/reinforce skills already taught in new situations, thus avoiding repetitious drill and promoting transfer of the skill to many areas
- flexibility in responding to student interests and needs. Learning will be facilitated when students see activities as being worthwhile and meaningful
- opportunity to use a wide variety of activities, media and resources
- opportunity to develop strategies and skills in problem solving and decision making through investigations that reflect real issues and problems present in the local community.

While the thematic approach is suggested in Mathematics 16, skills-based lessons or units should be provided when specific deficiencies are noted. For example, the need to develop and refine estimation skills may become obvious as students perform computations within a particular theme. Instruction and practice in estimating might then be provided through a separate sub-unit that focuses entirely on developing estimation strategies. Teachers are encouraged to determine which skills should be removed from the thematic context and taught or reinforced in a more focused manner.

Four themes that assure coverage of the required concepts, skills and attitudes have been provided in the *Teacher Resource Manual*. At the local level, teachers may wish to develop additional themes that expand upon and reinforce learning objectives.

INTERPERSONAL SKILLS AND THE SOCIAL SPHERE

High school represents a transitional stage of life for students as they develop the ability to use adult skills. For some students it may be a time of uncertainty as they strive for independence, yet at the same time experience pressures from parents, peers, media and themselves. Response to these pressures may be inconsistent as students continue to develop reflective awareness and the ability to analyze their emotions.

Although schools are not the sole influences on the student's emotional, social and ethical behaviour, the instructional program does affect interpersonal learning.

The Goals of Secondary Education directly state the importance of affective and interpersonal goals when they indicate that students should:

- learn about themselves and develop positive, realistic self-images
- develop constructive relationships with others based on respect, trust, cooperation, consideration and caring as one aspect of moral and ethical behaviour.

Students will vary in their emotional/social development and their ability to cope with personal problems. Behaviours viewed as "problematic" are often simply an indication of a student's lack of sophistication in using adult skills. Classroom instruction must provide a variety of approaches that will encourage students to reflect upon their responses in social situations and to develop productive interpersonal skills. The guidelines that follow have been adapted from Alberta Education's monograph *Students' Interactions: The Social Sphere* (1988), and are intended to foster affective, interpersonal and moral learning within the classroom:

- Students imitate and thereby implicitly learn to deal with emotions, other people and ethical issues by observing the consistency of adult behaviour. Therefore, it is important that appropriate behaviour be modelled for students. When teachers are respectful of students' dignity, students will respond positively.
- Encourage students to express their opinions and feelings; to ask questions and to accept emotions as they occur in day-to-day life. Through mediated learning (see "Instructional Mediation"), encourage students to examine emotional responses from different frames of reference, and to organize and interpret their own responses as well as the responses of others.
- Provide students with supportive comments, guidance and genuine expressions of concern. Set expectations that are firm and fair, and then believe in the students' ability to meet these expectations and do well. Develop "working agreements" to help tasks flow smoothly, and to ensure that students understand the nature of the instructional tasks they are asked to perform.
- Ensure that classroom management practices and rules are known, upheld, moderate in nature, negotiable, and consistently applied. Responding to the harmful or unjust effects/consequences of a moral transgression is more effective than reference to broken rules or unfulfilled social conventions.
- Recognize that experiential learning is a particularly effective vehicle for teaching interpersonal skills. Although some learning may occur through listening and reading, one best learns to live with other people by living with other people. Cooperative learning techniques are especially useful where students are actively involved in lessons linked to their own needs, interests or experiences.

- Be aware that although teenagers deal with a number of issues, these issues usually come into focus at different times. The issues are not so interdependent that the solution of one of them requires prior solution of others. Help students to integrate various aspects of their lives by encouraging them to recognize how various problems/issues/solutions are often interrelated.

- Assist students to learn skills that are appropriate in differing contexts. Although some students are described as "lacking in social skills", socially maladapted students do not necessarily lack either skills or social involvement; rather, they use inappropriate skills in particular contexts. When directly teaching interpersonal skills, be as concrete as possible, and "build bridges" by linking situations with appropriate actions and behaviours.

- Assist students to focus on the need for a system of shared conventions. As students affirm the social system of conventions, they will view conventions more positively and will become less disruptive in their behaviour.

- Encourage students to interpret and evaluate ethical issues presented to them. Provide opportunities for open discussion and debate, where students interact with their peers. Discuss issues that are "real" to the student.

- Provide students with practical strategies for resolving interpersonal conflict. A framework for social problem solving is provided in the *Teacher Resource Manual*. This framework uses a problem-solving approach in helping students to identify:
  - reasons for the difficulty
  - strategies to avoid the conflict another time.

Student development in the affective and interpersonal domains has been addressed in this curriculum through attitudinal learning objectives that accompany each cluster of concepts and skills in the Program of Studies. Instruction must include a balance of approaches appropriate to student development in each domain, as delivery of isolated content will not ensure the formation of desired attitudes. The *Teacher Resource Manual* provides additional strategies that facilitate attitudinal development within the context of themes suggested in this program.

REQUIRED AND ELECTIVE COMPONENTS

The required component of the Mathematics 16 Program reflects the concepts, skills and attitudes that all students must acquire, and has been established on the basis of frequent task demands placed upon students in daily life. The Program of Studies outlines the required component of the program.

In keeping with the philosophy that concepts, skills and attitudes are more successfully learned if taught within relevant contexts, they have been embedded in four themes:

- Earning Money
- Budgeting and Banking
- Skills for the Consumer
- Math in the Workplace.

Study of the topics outlined in these themes will ensure coverage of the required component.

The elective component of Mathematics 16 permits the teacher to:

- **remediate** or **reinforce** skills from the required component as necessary to achieve student mastery. Focused instruction on some skills may be appropriate if the "skills in context" approach has not been effective
- **extend** or **enrich** the program by way of introducing additional concepts and skills considered appropriate to student interest and need.

Students' abilities, interests and needs will largely determine how the elective time will be used. The elective component provides opportunities for varying organizational and instructional strategies that facilitate learning and that are appropriate to each student's developmental stage and preferred way of learning.

The instructional time for Mathematics 16 should be apportioned:

- 80% Required
- 20% Elective.

OVERVIEW OF THEMES COVERING THE REQUIRED COMPONENT

The required concepts, skills and attitudes identified in the Program of Studies have been integrated into thematic units of instruction provided in the *Teacher Resource Manual*. Although study of each theme involves the use of a variety of skills, themes vary in the degree to which they lend themselves to particular strands. The matrix indicates the nature of the skills that are emphasized throughout each theme.

	Number Systems and Operations	Ratio, Proportion and Percent	Algebra	Geometry and Measurement	Data Interpretation and Display
EARNING MONEY	X	X	X	X	X
BUDGETING AND BANKING	X	X	X		X
SKILLS FOR THE CONSUMER	X	X	X	X	X
MATH IN THE WORKPLACE		X	X	X	X

SUGGESTIONS FOR THE ELECTIVE COMPONENT

Several factors should be considered in choosing content for the elective component:

- student ability/interest/needs
- curriculum objectives (adequacy in covering basic skills)
- availability of suitable learning resources.

For some students, the elective component may be used to provide additional instructional time for study of the four themes that support the required portion of the program (i.e., extension and remediation). In other instances, however, teachers may wish to develop enrichment themes using the elective component of the program. Themes of this nature might relate to:

- mathematical competencies required in occupational courses
- a life experience or student interest topic
- a mathematical skills unit.

PLANNING

GENERAL COURSE PLANNING

Themes and their subsequent concepts, skills and attitudes may be sequenced at the teacher's discretion. Program planning should take into consideration the sequential and developmental nature of learning in mathematics as well as students' interests, abilities, and preferred ways of approaching tasks. Four themes that cover the required components of the course have been provided in the *Teacher Resource Manual*. These themes are intended to be descriptive rather than prescriptive. Teachers may choose to modify these themes or replace them with other locally developed material in addressing individual student needs.

Through cooperative conferencing, teachers may find that students are required to use certain mathematics-related competencies in other courses before they are learned in mathematics class. Joint planning and negotiation with teachers of other courses will be required in establishing an integrated program that places consistent expectations upon the student. (For example, students may benefit from study of the theme "Math in the Workplace" early in the program as this theme addresses tne skills frequently demanded of students in their occupational courses.)

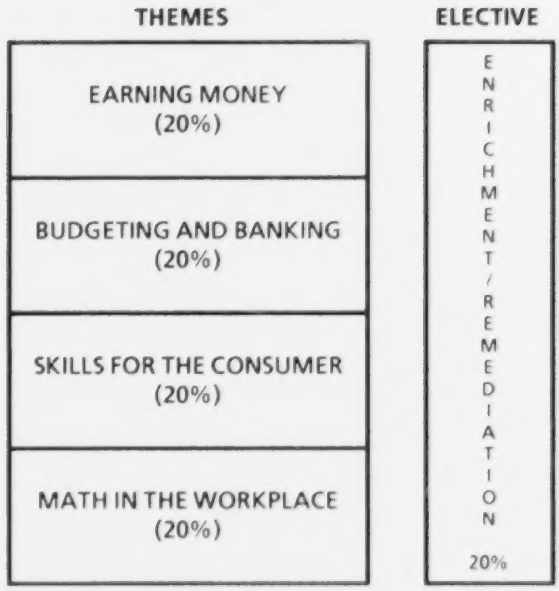
Program planning should emphasize the use of appropriate strategies for problem solving, as well as the development of computational facility and estimation skills. An understanding and appreciation of the "tools" of technology (e.g., the calculator and computer) should be developed through first-hand interaction with these technologies. The strategies and activities suggested throughout both the *Program of Studies/Curriculum Guide* and *Teacher Resource Manual* are numerous, but by no means exhaustive. Teacher use of these ideas will depend upon their appropriateness in meeting individual student's needs. **Be prepared to add, delete, and modify activities in adapting a theme to the particular circumstances of the classroom and student.**

TIME ALLOCATIONS

As a 3-credit course, Mathematics 16 is designed to be taught through 75 hours of instruction. In meeting student needs, however, local jurisdictions may find it desirable to allocate additional time for instruction in I.O.P. mathematics. (For example, schools may offer this 3-credit course through 125 hours of instruction.)

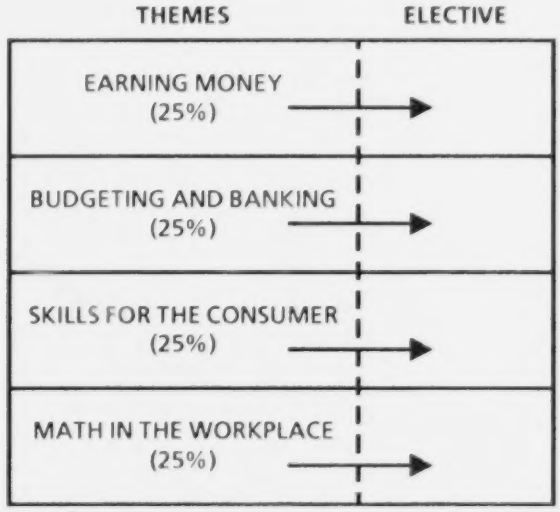
Minimum time allocations are recommended for the delivery of themes provided in the *Teacher Resource Manual*. These recommendations are intended to provide guidance for the teacher.

Earning Money	20%
Budgeting and Banking	20%
Skills for the Consumer	20%
Math in the Workplace	20%
Required Time	80%
Elective Time	20%
Total	100%



If student needs suggest that the elective component be used to provide additional instructional time for study of the four themes that support the required component of the program (i.e., extension and remediation), time allocations for each theme might be increased to those indicated below.

Earning Money	25%
Budgeting and Banking	25%
Skills for the Consumer	25%
Math in the Workplace	25%
Total	100%



Teachers may find it desirable to plan programs using time allocations that fall between those described in these two alternatives.

COMMUNITY PARTNERSHIPS

The mathematics program must enable students to recognize the relevance of computational competence and problem-solving skills in daily life experiences within the home, community and work environments. Within this context, students will be expected to demonstrate competencies that will enable them to:

- apply mathematical concepts and skills to practical situations
- set goals, solve problems and make informed decisions
- prepare for a chosen occupation or career.

Community partnerships (i.e., community-based learning experiences) will foster an appreciation of mathematics for its usefulness and relevance, and will assist students to transfer specific mathematical concepts and skills to more generalized situations in everyday life and the world of work. Guest speakers, field trips, job shadowing and mentorship are but a few examples of inviting members of your community into the class, or having students involved in the community by way of meaningful activity linked to mathematics.

Relevant community partnerships for Mathematics 16 may include:

- inviting guest speakers from local government, business and industry to discuss topics related to those studied in thematic investigations
- visiting local business, industry, and recreational facilities for first-hand observation and real life experience in areas related to the themes studied
- taking a walk into the community in search of applications made of the concepts and skills being studied (e.g., advertising billboards, information/direction signs, tools/units of measure, presence of geometric form)
- visiting the local TV station or newspaper plant in order to gather information related to mathematics and the media
- investigating career and employment opportunities in areas that require specific mathematical competencies (e.g., job shadowing, mock employment interviews).

Community agencies and groups that may provide meaningful contributions to the mathematics program include:

- businesses involved in retail sales and the promotion of consumer products/services (e.g., supermarkets, drug stores, department stores, hardware stores)
- financial institutions (e.g., banks, credit unions, loan companies)
- government agencies (e.g, Consumer and Corporate Affairs, Labour and Employment Standards, Career Development and Employment, Revenue Canada, Unemployment Insurance Commission, Workers' Compensation Board)
- businesses and industries offering potential career opportunities (e.g., construction, decorating and repair service, retail sales and marketing, food production and service).

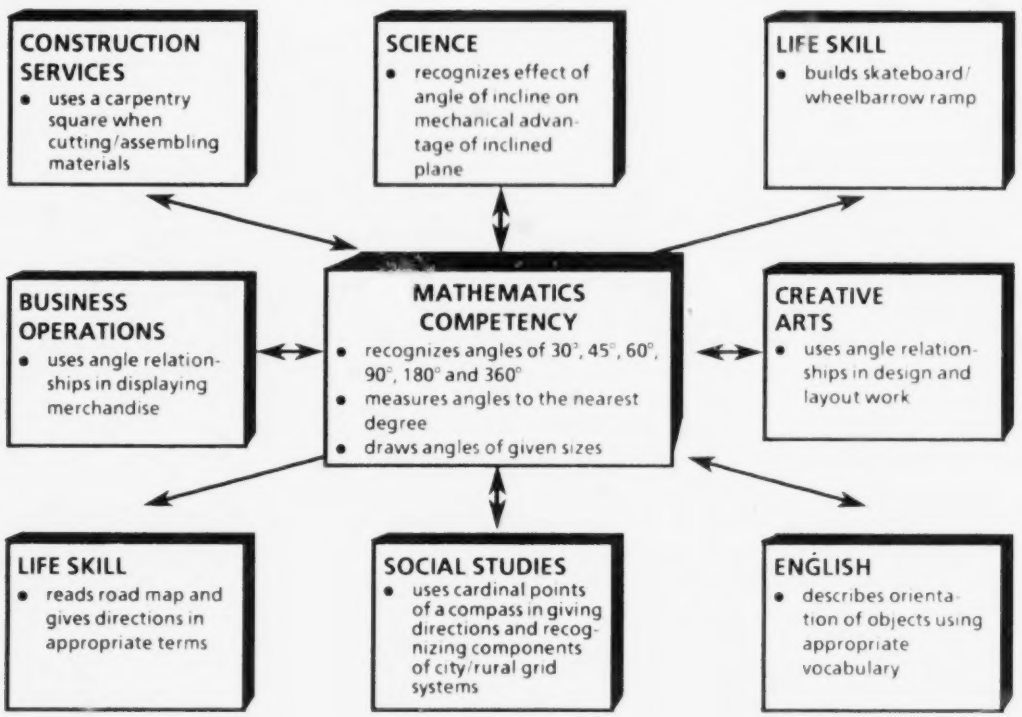
Additional suggestions for community partnerships are provided within the themes outlined in the *Teacher Resource Manual*. Activities have been selected on the basis of their value in furthering the objectives of each theme, as well as in reducing classroom barriers to real life experience.

CURRICULAR INTEGRATION

The mathematics program facilitates the development of effective problem-solving and decision-making behaviours by providing learning experiences that reflect and build upon the demands of home and school. Program content is developed within the context of its application to daily living, the occupational program and other academic disciplines. The integration of curricular content with contexts for application facilitates the transfer of specific concepts, skills and attitudes to more generalized situations, and serves as a motivating factor when students recognize the relevance of their learning experiences.

Teachers must be familiar with the mathematical competencies required of students in everyday life, the occupational program and other academic disciplines. Cooperative planning and conferencing with other teachers is central to understanding the contexts in which basic skills are used, and will assist teachers in providing practical and relevant learning experiences that facilitate the transfer of knowledge and skill.

EXAMPLES OF CURRICULAR INTEGRATION BASED ON THE MATHEMATICAL COMPETENCY OF ANGLE MEASUREMENT


CONSTRUCTION SERVICES
• uses a carpentry square when cutting/assembling materials
SCIENCE
• recognizes effect of angle of incline on mechanical advantage of inclined plane
LIFE SKILL
• builds skateboard/ wheelbarrow ramp
BUSINESS OPERATIONS
• uses angle relationships in displaying merchandise
MATHEMATICS COMPETENCY
• recognizes angles of 30°, 45°, 60°, 90°, 180° and 360°
• measures angles to the nearest degree
• draws angles of given sizes
CREATIVE ARTS
• uses angle relationships in design and layout work
LIFE SKILL
• reads road map and gives directions in appropriate terms
SOCIAL STUDIES
• uses cardinal points of a compass in giving directions and recognizing components of city/rural grid systems
ENGLISH
• describes orientation of objects using appropriate vocabulary

Angle concepts and skills are developed in mathematics and related to their applications in other areas. Subject areas across the curriculum will maintain and reinforce specific concepts and skills related to angles while they are being used within each discipline. While in this instance application is shown in all subject areas, some skills may have a more limited base for application.

The Program of Studies/Presentation of Content section of this document facilitates curricular integration by relating mathematical competencies to daily life skills (Column 2) and to other applications across the curriculum (Column 3). The *Teacher Resource Manual* provides further opportunities for developing mathematical competencies within the context of their application to other subject areas (see "Integration Activities").

PLANNING AN INTEGRATED UNIT OF INSTRUCTION

A variety of factors needs to be considered when expanding upon an existing theme or developing a new thematic unit. The guidelines that follow provide structure and direction for developing units of instruction.

1. Identify a possible theme, based on:
   - curriculum objectives (adequacy in covering basic skills)
   - student needs/interests/abilities
   - availability of suitable learning resources.

2. Develop a purpose for the theme. Include:
   - thematic objectives
   - a checklist of concepts, skills and attitudes that lend themselves to the theme. Identify those skills that may need some focused/direct teaching.

3. Consider suitable resources:
   - books, pamphlets, monographs
   - computer software and other technology
   - resources from the occupational program
   - community contacts
   - newspaper and magazine articles
   - other resources to which students may have access.

4. Design activities:
   - allocate activities to the purposes developed in Step 2
   - develop a checklist of process objectives
   - break activities into lessons with general objectives
   - sequence the lessons.

5. Develop ongoing strategies to build community partnerships into your theme:
   - field trips
   - guest speakers.

6. Plan for evaluation:
   - student evaluation
   - teacher's ongoing (formative) and summative evaluation.

7. Share the unit:
   - other teachers need access to good work!
   - keep notes, reflect, expand and rework the unit each time it is used
   - as others experiment and your unit enlarges, develop a mechanism for evaluating activities with different classes of students.

LEARNING RESOURCES FOR MATHEMATICS 16

STUDENT RESOURCES

BASIC LEARNING RESOURCE

The learning resource listed below addresses the majority of the goals and learning objectives identified in this *Program of Studies/Curriculum Guide*.

Carli, Enzo, et al. *Consumer and Career Mathematics, Canadian Second Edition*. Agincourt, Ontario: Gage Educational Publishing Company, 1991.

SUPPORT LEARNING RESOURCE

The learning resource listed below assists in addressing some of the learning objectives identified in this *Program of Studies/Curriculum Guide*. Multiple copies (but not necessarily class sets) may be desired.

Cutting, Robert, et al. *Math You Need, Book 2*. Scarborough, Ontario: Nelson Canada, A Division of International Thomson Limited, 1982.

OTHER LEARNING RESOURCES

A listing of other learning resources identified as useful in implementing the Mathematics 16 Program, but which have not undergone the standard review procedures of Alberta Education is provided in the *Teacher Resource Manual*. The responsibility for evaluating these resources prior to selection and use rests with the local jurisdiction.

TEACHER RESOURCES

The teacher support publication listed below is designed to assist classroom teachers to implement the Integrated Occupational Mathematics 16 Program. The publication provides teaching strategies and sample activities that structure the development of concepts, skills and attitudes as outlined in the Program of Studies, and supports thematic instruction in four areas described as covering the required components of the program.

Alberta Education. *Mathematics 16 Teacher Resource Manual*. Edmonton, Alberta: Alberta Education, 1990.

Other teacher resources that address major goals and learning objectives identified in this *Program of Studies/Curriculum Guide* include:

Charles, R. et al. *How to Evaluate Progress in Problem Solving*. Reston, Virginia: The National Council of Teachers of Mathematics, 1987.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Virginia: The National Council of Teachers of Mathematics, 1989.

A listing of additional teacher resources that may be useful in implementing the Mathematics 16 Program are identified in the *Teacher Resource Manual*.

TECHNOLOGY AND MEDIA

SUPPORT LEARNING RESOURCES

The computer software programs listed below assist in addressing some of the learning objectives identified in this *Program of Studies/Curriculum Guide*.

Title:	*Math Strategies: Problem Solving*
By:	Science Research Associated (Canada) Ltd. 707 Gordon Baker Road Willowdale, Ontario M2H 2S6
Components:	2 disks, teacher's guide (28 pp.), 20 student texts (108 pp.)
Objectives:	This program is designed to provide instruction and practise in solving multiple-step problems using four problem-solving strategies: simplifying a problem, breaking a problem into parts, identifying needed additional information, and making a model of the problem.

Title:	*Money Manager*
By:	Computer Age Education Inc. 52256 Wisconsin Avenue N.W., Suite 601 Washington, D.C. 20015
Components:	1 disk, teacher's guide (11 pp.), user's guide (4 pp.)
Objectives:	This program is a simulation designed to help the student develop and practise skills in financial management. The goal is to earn the most financial points possible over a simulated eight-week period.

An annotated list of additional computer software available in mathematics that may also address components of the program is provided in Alberta Education's catalogue of *Computer Courseware Evaluations* (Curriculum Support Branch, Student Programs and Evaluation Division, Alberta Education, 1986) and annual supplements.

OTHER LEARNING RESOURCES

A listing of other technology and media resources (e.g., films, videos, software, kits, pamphlets) identified as useful in implementing the Mathematics 16 Program, but which have not undergone the standard review procedures of Alberta Education is provided in the *Teacher Resource Manual*. The responsibility for evaluating these resources prior to their selection and use rests with the local jurisdiction.

ACCESS NETWORK

ACCESS offers a variety of resources and services to teachers. For a nominal dubbing and tape fee, teachers may have ACCESS audio and video library tapes copied. ACCESS also offers a service called "Night Owl Dubbing". This allows educators to tape late night educational programs directly from their own televisions.

ACCESS publishes both an *Audio-Visual Catalogue* and a comprehensive schedule of programming, available on request.

For additional information, contact ACCESS NETWORK, Media Resource Centre, 295 Midpark Way SE, Calgary, Alberta, T2X 2A8 (from outside of Calgary, telephone toll free, 1-800-352-8293; in Calgary, telephone 256-1100).

REGIONAL RESOURCE LIBRARIES

Films and videos are available for loan through the five centres listed below. In some instances, computer software is also loaned. Catalogues of holdings are available upon request.

Zone I

Zone One Regional Film Centre
P.O. Box 6536 / 10020 – 101 Street
Peace River, Alberta
T8S 1S3
Telephone: 624-3187

Zones II and III

Central Alberta Media Service (CAMS)
2017 Brentwood Boulevard
Sherwood Park, Alberta
T8A 0X2
Telephone: 464-5540 / 467-8896

Zone IV

Alberta Central Regional Education Services (ACRES)
County of Lacombe No. 14
Box 3220 / 5140 – 49 Street
Lacombe, Alberta
T0C 1S0
Telephone: 782-5730

Zone V

South Central Alberta Film Federation (SCAFF)
Westmount School
Box 90 / Wheatland Trail
Strathmore, Alberta
T0J 2H0
Telephone: 934-5028

Zone VI

Southern Alberta Regional Film Centre (SARFC)
c/o McNally School
P.O. Box 845
Lethbridge, Alberta
T1J 3Z8
Telephone: 320-7807

METHODOLOGY

OVERCOMING MATHEMATICS ANXIETY

Anxiety about mathematics may represent a major barrier to learning for many students. Previous experiences, perceptions of the value of mathematics, and the fear of being wrong or making a mistake are all factors which may cause the student to look upon mathematics class as a place where personal inabilities are exposed. To assist students in dealing with their anxieties, a supportive environment should be provided where meaningful learning activities ensure that each student experiences success. The program must be varied in materials, content and instructional method in order to meet individual student interests and abilities.

A supportive classroom environment that decreases anxiety levels may be fostered through use of the following strategies:

- Begin instruction well below the "frustration" level. Providing "warm-up" activities enables the anxious learner to build on previous success. Learning activities should correspond to the student's current operational level rather than the level at which it is considered the student should be working.

- Relate mathematical concepts and skills to the learner's own experiences. Ensure that the terminology used is part of the student's working vocabulary. Focus attention on what students consider interesting or important. Familiar problem situations will "make sense" to the student more readily than unfamiliar situations.

- Avoid unnecessary tension in the classroom by being patient, receptive and understanding. Ensure that students recognize that making a mistake is acceptable, and that such occurrences provide valuable learning experiences. Urge students to ask questions, and to accept that "there is no such thing as a stupid question". Remember that an offhand remark about a problem being "easy" may be interpreted by the student as "if it's easy and I'm confused, then I must be stupid". Recognize and accept alternative solutions that students may devise.

- Provide abundant opportunities for learning at the concrete level. Tactile experiences with base-ten blocks, geoboards, fraction pieces and cuisenaire rods will assist concept formation and application to problem situations. Encourage students to verbalize and discuss relationships discovered. Assist students to translate relationships discovered in the concrete to the abstract symbols used in mathematics.

- Match the reading level of resources and materials used to that of the students. Steps should be taken to ensure that reading deficiencies do not prevent students from learning mathematics. Additional strategies are provided in a later section entitled "Use of Language in Learning Mathematics".

- Group for instruction through a variety of organizational patterns that will facilitate meeting the needs of individual students. Ensure that grouping patterns remain flexible, allowing student movement from group to group. Include some learning activities that involve the whole group, small groups, and one-to-one (teacher to student and/or student to student).

- Use a variety of assessment and evaluation strategies. While paper-and-pencil tests are an effective way of evaluating some learning outcomes, their use does not always enable students to demonstrate their strengths and provide for successful experiences. In such cases, provide students with opportunities for retesting if necessary. Effective evaluation should draw upon information gathered from a number of sources, including teacher observation, student demonstration and project work. Additional strategies for student evaluation are provided in the "Evaluation" section of the document.

- Recognize that while most anxiety symptoms are brought on by natural apprehension associated with new or challenging situations, there are other displays of anxiety that may well be related to deeper personal or social problems. Personal interviews with students may assist in identifying such instances, and may also establish a need for consultation (or referral) with counsellors who have expertise in dealing with such problems.

EXPERIENTIAL APPROACHES TO MATHEMATICS

Student learning styles and developmental levels suggest a multisensory approach to learning, involving practical situations that may be experienced or simulated by the student. Investigations should be chosen on the basis of their familiarity and relevance to students. Abstract concepts can best be developed through a variety of tactile experiences involving manipulative and visual materials. An experiential approach suggests that instruction in mathematics include:

- active student involvement
- activities that involve the concrete, transitional and formal levels of cognition
- activities that address individual developmental levels
- deliberate observation and questioning that promote thinking.

Practical experiences and everyday situations are referenced throughout "themes" provided in the *Teacher Resource Manual*. Each theme relates the concepts and skills being studied to practical situations that are familiar to the student. The suggested activities will assist students to transfer skills that are learned to familiar experiences in daily living. The "community partnership" suggestions provided in each theme will assist in providing relevant learning experiences for students in the program.

The use of manipulative and visual materials in developing abstract concepts and skills is discussed in both this document and the *Teacher Resource Manual*. Every student should have direct access to the use of manipulatives while developing a new concept. Cautionary notes have been provided where it is anticipated there may be a lack of congruence between cognitive demands of the curriculum and the developmental level of students (e.g., fractions, ratio, percent, two- and three-dimensional geometry). In these instances, the use of manipulative materials becomes increasingly important in providing students with meaningful learning experiences. Manipulative materials need not be expensive, and in most instances can be improvised through the use of other materials that are readily available (blackline masters, strips of coloured paper, checkers, empty household containers, and so on).

Encourage students to observe, verbalize and discuss the relationships they investigate, and to eventually translate relationships into the abstract symbols of mathematics. While the purpose of manipulatives is to help students understand and remember, there comes a time when each student should become efficient in making application of concepts in their abstract form. It is important, therefore, to supplement/alternate the use of manipulative materials with other more abstract learning strategies and activities.

PROBLEM SOLVING

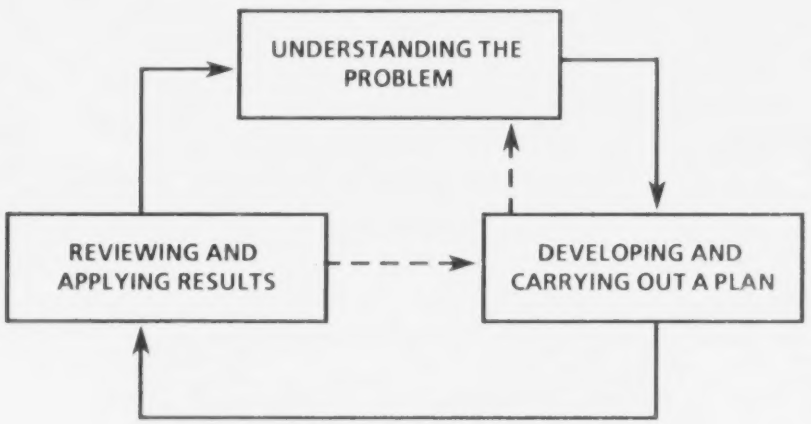
Learning to solve problems is a major goal of the mathematics program. Today's rapidly changing technological society demands that students are able to apply the mathematical skills learned to new and unfamiliar problem situations in life. Practice in finding answers to routine word problems in mathematics will not provide students with the problem-solving skills they require. Instead, students must acquire strategies for solving problems that foster the development of critical and creative thinking skills, and be given ample opportunity to apply the strategies and skills they acquire to a wide variety of problem situations in everyday life. The mathematics program must provide opportunities for students to apply knowledge, skill and experience in new and challenging situations where:

- no readily apparent solution or means to the solution is evident
- a person can be temporarily perplexed
- there may be no answer, a single answer, or many answers
- personal and societal factors are involved, as well as mathematical competencies.

Problem solving should not be viewed as an isolated activity. Strategies and skills appropriate to problem solving must be integrated and applied throughout all themes and strands of the curriculum. While a detailed framework for problem solving is outlined on the following pages, additional information can be obtained by referring to Alberta Education's monographs *Problem-Solving Challenge for Mathematics* (1985) and *Problem Solving in Mathematics: Focus for the Future* (1987). Classroom activities that contribute to an understanding of the problem-solving process are provided in the *Teacher Resource Manual*.

A FRAMEWORK FOR PROBLEM SOLVING

A framework for problem solving is diagrammed below. Although the framework provides an overall structure that can help students to understand the processes used in solving problems, it should not be interpreted as a series of fixed and rigid stages and strategies. There are many ways to approach and solve problems, and the sequence of steps used will depend upon individual problems and individual students. Students may not always go through the steps in the order suggested, nor even use all steps in some problem situations. Nevertheless, an understanding of the steps as well as individual skills that can be used at each stage of the process will increase the student's repertoire of strategies that can be brought to bear on a problem.



The framework for solving problems in mathematics parallels the framework used for solving problems in other subject areas within the Integrated Occupational Program, and establishes a general "plan for action" that can guide students through a broad range of investigative activities (e.g., problem solving, inquiry, decision making, library research). As students become familiar with the framework and discern similarities among the actions taken in different kinds of investigation, they will be more likely to transfer skills learned in specific contexts to other practical situations where methods and outcomes may be uncertain. Teachers should encourage students to recognize how the framework for solving problems in mathematics provides a general plan for action that is also effective in guiding investigations in English, science, social studies and the occupational program, and how specific skills used at each stage within the framework will vary according to the nature of the problem or investigation.

USING THE PROBLEM-SOLVING FRAMEWORK

Performance in problem solving can be enhanced by making students conscious of their thought process (i.e., metacognitive awareness). Students need to become aware of and discuss how they think in order to become more strategic in their learning repertoires, and to monitor their problem-solving efforts (e.g., check their answers when in doubt). Teachers can foster this awareness by thinking out loud as they solve problems, by asking questions, and by having students identify the strategies and processes used when solving problems. Teacher modelling of appropriate steps and strategies for solving problems will provide a structure that facilitates the development of effective problem-solving skills.

Various stages and steps in the problem-solving process are clarified in the paragraphs that follow.

UNDERSTANDING THE PROBLEM

During this stage of the problem-solving process, students must be encouraged to think about and interpret the problem situation. Teachers can assist students to focus their attention on information and conditions set in the problem by asking appropriate chains of questions. Teachers can model and explicitly teach strategies that may be used by students in developing an understanding of a problem situation.

DEVELOPING AND CARRYING OUT A PLAN

In this stage, students should plan strategies and then use these strategies to actually solve the problem. As students may lack the "strategic repertoire" required to develop a problem-solving plan, it may be necessary to explicitly teach various strategies appropriate to specific problem situations. Emphasize that there are often strategies other than computation that can be used effectively to solve the problem. Once appropriate strategies have been planned, the student simply "carries out the plan" to arrive at a solution.

REVIEWING AND APPLYING RESULTS

This stage encourages students to assess the effectiveness of their solution, and to consider the accuracy of their results. Students should be encouraged to relate answers to the question in the problem in order to verify that the problem has indeed been solved. By evaluating strategies that have been used, students will become aware of their appropriateness and of other strategies that might also be used. Through discussion and reflection, encourage students to generalize and apply their results to related situations.

Appropriate strategies that may assist students to understand a problem, to develop and carry out a problem-solving plan, and to review and apply the results of problem solving are outlined in further detail within the "Program of Studies/Presentation of Content" section of this document.

Questioning and modelling techniques that foster metacognitive awareness and the development of effective problem-solving strategies are provided in the "Instructional Mediation" section of this document.

THE ROLE OF THE TEACHER IN PROBLEM SOLVING

Critical to the development of skills in problem solving is the attitude with which students approach the task. The development of appropriate attitudes must be nurtured through an atmosphere that fosters flexibility and acceptance. Students must learn to accept and appreciate that being perplexed and unsure is often normal when first encountering a challenging situation, and to take risks in the development of particular problem-solving strategies. The guidelines that follow may assist teachers in planning effective problem-solving activities.

- Create a positive classroom atmosphere that facilitates the development of appropriate attitudes and a desire to solve problems by:
  - encouraging students to be creative, and to generate their own ideas and approaches to problem solving
  - providing adequate support and assistance to students, but not solving their problems for them
  - being willing to accept unconventional solutions, more than one solution, or no solution, where appropriate
  - challenging students to think critically and justify strategies and solutions
  - being enthusiastic and capable of recognizing the students' desire and perseverance to solve problems
  - providing appropriate questions and modelling for students.

- Share the problem-solving framework and strategies with all students. The model provides structure to the overall process, as well as specific strategies that might be used at each stage of the process. Familiarity with the model will increase the students' strategic repertoire.

- Relate the problem-solving model used in mathematics to the models used for solving problems in science, social studies, English and the occupational courses. Students should recognize how the models are similar in the overall structure they provide for problem solving, and how the models may differ in terms of individual strategies appropriate to each stage of problem solving.

- Keep the framework and strategies for problem solving flexible and tentative. While useful in the support and structure they provide, students should be encouraged to be creative and experimental in their approach to problem solving.

- Select problem-solving activities that emerge from real life situations. Ensure that problems are relevant to the interests and experiences of students, and that cognitive demands of the problems match developmental levels of students.

- Modify and vary the approach used so as to ensure interest, participation, and some degree of success by all students. Most students have an inherent desire to accept the challenge provided by a problem.

USE OF TECHNOLOGY

THE CALCULATOR

The calculator has become an integral part of our way of life. Students must become proficient and discerning in its use as an aid to computation and problem solving. **All students shall have calculators readily available for use throughout the program.**

Effective use of the calculator requires an understanding of place value and the ability to judge the reasonableness of the results of calculations. Skills in estimation and mental arithmetic become increasingly important in enabling students to anticipate and verify calculator results. These experiences will be more effective in developing number sense and cognitive process than long and tedious computations with paper and pencil. Competence in estimation and mental arithmetic can be expected to improve through frequent use of these skills while using a calculator. By using calculators appropriately, time previously spent on tedious computation can be used to develop cognitive process and problem-solving skills.

The guidelines that follow are intended to provide a classroom structure that facilitates appropriate use of the calculator as an instructional and computational tool.

- The calculator does not reduce the need for basic arithmetic skills. **Effective use of the calculator requires that an understanding of place value, basic number skills, and arithmetical operations is emphasized throughout the instructional program.**
- There will always be a need to possess limited paper-and-pencil computational skills. **For most students, the calculator should be used after the mathematical concepts and algorithms are understood.** Students who simply cannot master their basic number skills or who have persistent problems with computation should use calculators more extensively.
- The calculator should always be used in situations where long and/or extended computations are required.
- **When using the calculator in problem situations, place emphasis on estimation and mental arithmetic**, on proper documentation of numbers and operations used, and on the reasonableness of answers.
- Do not assume that students understand how to use a calculator. **Be prepared to teach students how and when to use a calculator properly.** Remember, however, that creating a situation just for the sake of using calculators will not develop an understanding of their appropriate use.

THE COMPUTER

Current demands of society place emphasis not only on the ability to communicate through the written word, but also on the skills of interacting with computer technology. The mathematics program shall provide opportunities for students to demonstrate an understanding of:

- the basic operation of computers, by
  - identifying major parts of a computer
  - distinguishing between hardware and software
  - recognizing that computers get their instructions from a program written to accomplish a specific task.

- the basic capabilities, applications and limitations of computers, by
  - recognizing that computers are best suited to tasks requiring speed, accuracy, repeated operations and the processing of large amounts of data
  - identifying major areas in society where computers are used and the tasks performed by computers in these areas
  - recognizing tasks that computers cannot accomplish.

To the extent that facilities and equipment are available, the mathematics program should also provide opportunities for students to:

- work independently with prepared software
- enter, run and modify simple programs that have been written for particular purposes.

These experiences will support concept and skill development, while at the same time enable students to interact with computers and gain first-hand knowledge of their versatility and limitations. Computer programs and software should be selected on the basis of their effectiveness in developing mathematical concepts and problem-solving skills, as well as in providing for drill and practice.

Computer software that supports the development of specific goals and objectives within Mathematics 16 has been identified in another section of this document (see "Learning Resources for Mathematics 16"). The *Teacher Resource Manual* provides suggestions on how these programs, as well as other computer-oriented activities, might be used in regular classroom instruction.

COMPUTATIONAL FACILITY AND ESTIMATION

The availability and use of technology has caused emphasis in the skills required for computational competence to change over the last decade. Computational facility includes more than the knowledge and skills required to perform paper-and-pencil computation with standard algorithms. While these skills are important, responsible participation at home and work also requires facility in performing **mental arithmetic**, in the application of **calculator skills**, and in applying strategies of **estimation**. Surveys show that mental computation and estimation are used in more than 80 percent of all practical problem-solving situations encountered outside the classroom. As important as each method of computation is in itself, even more important is the **understanding of when each strategy is most appropriately used** in everyday problem-solving situations.

A brief description of each method of computation is provided below. Strategies and activities that will develop students' ability to compute using each method are provided in the "Program of Studies/Presentation of Content" section of this document, as well as in the *Teacher Resource Manual*.

PAPER-AND-PENCIL COMPUTATION

Instruction in paper-and-pencil computation should emphasize place value and basic number skills prior to the application of algorithms. Concrete and visual material will assist in concept development. Students who have not experienced past success with standard algorithmic procedures may react more favourably and experience more success with less sophisticated forms of algorithms.

The learning of paper-and-pencil computation must focus on an understanding of process and de-emphasize calculation with large numbers. Addition and subtraction should generally include numbers with no more than four digits. Multiplication and division should be performed on numbers containing up to four digits, using multipliers/divisors of no more than two digits.

MENTAL ARITHMETIC

The ability to perform mental computations is frequently required of students in a variety of situations. In addition to providing a fast solution to many everyday problems, this skill facilitates the ability to estimate and approximate. Experience with mental computation fosters the development of number sense and enables students to gain confidence in their ability to compute answers.

Students should have knowledge of basic number skills, and be able to apply these skills to more sophisticated processes. Teachers are encouraged to identify mental arithmetic strategies that are most appropriate for their students and to schedule a period of time each week in which these skills can be taught and practised. Short drill and practice activities that become part of the daily routine will promote the development of mental arithmetic skills and foster a habit for their use. Ensure that competition in these activities is with self rather than others in order to avoid humiliation of those who find mental arithmetic difficult. Encourage students to share with other members of the class the personal strategies in mental arithmetic that they find useful.

CALCULATOR SKILLS

Use of the calculator enables students to investigate, see patterns and relationships, and more readily solve problems that require higher levels of thought. Many mathematical problems that would otherwise involve long and unmanageable calculations can be investigated with the calculator.

The use of a calculator does not reduce the need for mastery of basic number skills. Students must recognize incorrect answers obtained on the calculator that result from incorrect numbers being entered or operations entered in the wrong sequence. In order to distinguish a reasonable answer from an unreasonable one, students need to know how to compute using the four operations. Skills in estimation and mental arithmetic are necessary in enabling students to anticipate and verify results obtained on the calculator.

Guidelines that foster the development of calculator skills are provided in the "Use of Technology" section of this document.

ESTIMATION

Skill in estimation is necessary for effective problem solving and calculator use. Students need to be able to determine if a particular result is precise enough for the purpose at hand, and be alert to the reasonableness of computational results. More often than not, practical problems involve estimations rather than exact numbers. Estimation, not computation, gives answers to everyday questions like "Do I have enough cash to buy groceries?" and "How many hot dogs should I order for the party?"

Thinking skills and problem solving assume important roles in developing estimation skills. It should be recognized that the process of estimation:

- is performed mentally, usually without paper and pencil
- is done quickly
- produces answers that are not exact, but adequate for making necessary decisions
- reflects individual strategies and produces a variety of estimates as answers.

In order to be able to carry out rapid estimations, students must understand place value, have skill in single digit operations, be able to multiply and divide by powers of ten, and have facility in rounding whole numbers and decimals to the number of significant digits required by the situation. Students should develop useful estimation strategies, and apply these strategies to whole numbers, decimals, fractions, and percentages. Testing and evaluation should include assessment of the student's ability to estimate in a variety of practical situations.

USE OF LANGUAGE IN MATHEMATICS

The strands of language (listening, speaking, reading, writing, viewing) play an important part in the teaching and learning of mathematics. Not only is language a means of communication, but it is also part of the thinking process used to combine ideas, find relationships, ask questions and solve problems. Language development in mathematics should parallel its development in English class by the use of similar strategies.

A distinction should be made between skills that are used to understand mathematics as a technical language, and the use of general language skills in studying mathematics and investigating related problem situations. Instructional planning should include strategies that facilitate language development in both of these areas.

MATHEMATICS AS A TECHNICAL LANGUAGE

The precision of mathematical vocabulary and symbols is often a source of difficulty for students. The everyday meanings associated with words often interfere with an understanding of the specialized meanings words may have in a mathematical context. Each theme within the program should be analyzed in order to identify demands relative to vocabulary and symbolic content. The following questions are useful in planning lessons that provide an understanding of the technical vocabulary used.

- How does this vocabulary relate to everyday usage?
- How does it conflict with everyday usage?
- How does it relate to previously studied mathematical terms?

The role of experience in developing concepts must be recognized and accounted for in vocabulary development activities. Definitions may be developed as summarizing statements of ideas that have been understood, but should not be used as introductions to new vocabulary or symbols. The following instructional sequence can be used to foster meaningful vocabulary development, and ensure that each word or symbol introduced becomes part of the students' active or "working" vocabulary.

- Discuss real life examples from the students' environment where the word/symbol might be used.
- Simulate concrete and transitional models where the word/symbol might be appropriately used.
- Discuss and list distinguishing characteristics of the word/symbol.
- Record the word/symbol and its distinguishing characteristics (meaning) in a personal "mathematics glossary".

USING LANGUAGE TO UNDERSTAND MATHEMATICS

The role of language in the study of mathematics can be strengthened through the use of questions that probe for explanations rather than stimulate recall of information. Students should frequently be asked "Why?" and encouraged to use language in expressing their ideas, interpretations and answers to questions asked. Opportunities should be provided for students to engage in:

- writing activities that describe mathematical relationships/patterns investigated and the results of problem solving
- speaking and listening activities where thinking and reasoning skills are used with their peers
- reading and viewing activities that are required for problem solving and gathering content or information.

Students can be guided in their reading/viewing for content and information through a sequenced approach that includes the following steps:

- Decide on a purpose for reading/viewing. State the purpose as a question.
- Skim the whole section to get an idea of how it is organized.
- Notice various "aids" to reading/viewing that have been used in the material (e.g., headings, colour, bold print, charts).
- After reflecting on purpose, decide which sections need intensive study and which may be delayed or skipped.
- Engage in "active" reading/viewing of the material (e.g., ask questions, think of examples, rephrase in everyday language).

Problem solving necessitates quite a different kind of reading, wherein relationships and patterns often need to be identified and analyzed before solutions can be attempted. Cognitive demands placed upon the student in these situations can be a source of frustration. Instruction must provide support, encouragement and assistance. Model strategies that will guide and facilitate the development of reading skill in this area. The following steps are effective in structuring activities that facilitate the development of reading skills used in problem solving:

- Read rapidly to grasp the general idea.
- Read again to determine facts and their relationships.
  - What facts are given?
  - What facts are not given?
  - How does each fact influence other parts of the problem?
- Read again to check facts and relationships.
- Determine the steps required for a solution.
  - What do I know?
  - What do I need to know?
  - How can I use what I know to find out what I need to know?
- Provide a comprehension check by attempting to estimate an answer to the problem.
  - Considering all information, is this answer reasonable?

INSTRUCTIONAL MEDIATION

A great deal of recent research has focused on instructional mediation and "teacher talk" in the classroom. Instructional mediation is an interactive process wherein teachers refine their interpretation of tasks to students, as students construct their own interpretations of the tasks and processes being learned. This back and forth exchange stimulates the development of thinking skills by allowing both sides to contribute to a meaningful learning situation. Lectures, or one-sided explanations, rely on students to be "self-mediating" and to supply their own meaning to processes. Most students can benefit from further practice in this area.

Instructional mediation regulates the students' behaviour in terms of the use of strategies and approach to tasks. Emphasis on a strategic view of tasks will encourage students to become independent in the tasks they perform and the processes they use. For example, a strategy for attacking mathematics problems will enable more students to solve problems on their own. In addition, such a strategy enables the student to identify for the teacher at what point they need assistance if they are unable to solve the problem completely.

A further use of mediation is in developing the students' feeling of competency. Students need to see themselves as competent and able to do things. Students who feel competent, and who recognize their effort as being effective in learning, are more likely to be persistent in attempting new tasks that are difficult. On the other hand, students who require frequent praise for their effort come to have limited performance goals and are hesitant to engage in any task at which they cannot quickly become successful. Teachers can encourage students to extend their learning goals by focusing mediation on the role of effort and strategy in achieving success.

In creating a classroom environment that will stimulate strategic behaviour and thinking skills, the teacher can:

- identify, but not correct errors
- encourage students to correct their own errors
- pause and clarify, but not interrupt
- demand constant vocalization of student thought processes that are used, and model these and related thought processes to students
- encourage persistence through praise.

The mediation process can be enhanced through the use of appropriate "questioning techniques" and the "modelling" of complete processes and thought patterns. These instructional methods are described in the paragraphs that follow.

QUESTIONING TECHNIQUES

Questioning techniques include the use of chains of questions that lead students to discover their own answers. Question chains should begin with focus questions such as:

- What is the goal of this problem?
- What do we want to do?
- What are we looking for?
- What is stopping us?
- Who can we ask?
- Where can we look?
- What should we do first?
- What is this process called?

Once students have established the significant pieces of information necessary to solve the problem, the focus is expanded through questions like:

- What can we do?
- Have we got enough ideas yet?
- What is our next step?
- How will we do that?
- What is the relationship between . . . and . . .?
- How are . . . and . . . related?
- How can we simplify . . .?

When students have succeeded with the problem, ask questions that encourage students to review the process used and evaluate their work. These questions may include:

- Have we finished?
- How can we decide if . . . is a reasonable solution?
- How can we check this solution?
- Is there a better solution?
- How do you know that we have solved the problem?
- What are different ways we can approach the problem?
- How could we make our work easier another time?
- When might you use this process again?
- What should we tell others?

Questions that probe and prompt students to process information, rather than fixate at the simple recall level, will make students aware that they are expected to be actively involved in thinking processes. When students experience success with problems through use of the teacher's chain of questions, they will recognize the importance of the thought processes they have used, and be encouraged to develop related strategies that direct their efforts in other tasks.

COGNITIVE MODELLING

Cognitive modelling, as opposed to demonstrating, involves "talking through" a complete process in order to expose thinking processes to the student. While in a demonstration everything turns out as it should, modelling should include false starts, trouble spots, and having to deal with errors. This type of modelling requires teachers to express their thinking processes out loud so that students can see not only how the process is done, but also how difficulties and ambiguities are addressed. As the process is modelled, students should recognize that the teacher's approach may not necessarily be the "best" approach, but simply one way of tackling the problem.

While teachers often model when explaining processes and problems, knowledge and awareness of the value of modelling can serve to sharpen this process. Asking students where to begin in a problem situation makes a good starting point. If the reply is "I don't know!", an area of difficulty is determined. The mini processes of "understanding the problem" and "developing a plan" might then be modelled for students.

EVALUATION

Evaluation should be viewed as an ongoing part of the teaching and learning process, providing feedback to the student, the teacher, and the parent. Major functions served by the process of evaluation include:

- provision of feedback to the student relative to his or her success in the learning process. Students have difficulty in monitoring and regulating their learning behaviours, and require a great deal of external feedback as to their progress. Feedback and encouragement must be provided on a regular basis
- provision of information to the teacher concerning the appropriateness of learning goals and objectives, and the effectiveness of learning strategies and materials that have been used. Such information enables the teacher to modify the program as required with respect to pacing, learning resources, teaching methods or objectives
- provision of information to the parent regarding the student's progress. Where possible, reports to parents should be interpreted through an interview so that the implications of the evaluation are understood. While useful in communicating student progress to parents, the interview is also valuable in identifying individual needs that may be met through program planning and delivery.

Evaluation should serve diagnostic purposes in identifying student strengths and weaknesses, as well as summative needs in measuring overall growth. Because evaluation is an integral part of all aspects of the instructional process, information used in the evaluation of a student should be gathered from a variety of sources using a variety of methods. The evaluation program in mathematics must consist of more than paper-and-pencil tests. While such tests may be an effective way of evaluating the learning outcomes of specific computational skills and applications, an effective system of evaluation should include, to some degree, the use of all of the following sources of information:

- observation of attitudes and performance
- oral and written presentations of solutions to problems/mathematical applications
- personal interviews with students
- attitude scales
- project work
- feedback from parents
- teacher rating scales/checklists/inventories
- self-rating/self-marking
- peer marking
- records of previous achievement
- quizzes related to specific objectives
- diagnostic tests
- pre-tests and post-tests on topics or units
- suitable standardized examinations.

STRATEGIES FOR EFFECTIVE EVALUATION

Evaluation may, in the past, have been the process by which some students within the program were identified as "failures". These students will go to extreme measures to avoid being "tested" again. Absence from examinations; feigning an "I don't care anyway" attitude; or not giving their best effort so that the anticipated failure can be combatted with "I didn't try my best", are all common behaviours. It is therefore important to provide variation in assessment procedures that are used so as to draw upon students' strengths and provide for their success in the evaluation process.

The strategies provided here are intended to serve as guidelines to the teacher in developing a system of evaluation that will improve both student learning and the quality of the mathematics program offered to students.

- Provide frequent opportunities for students to practise the use of basic number skills, mental arithmetic and estimation. In addition to providing encouragement and feedback to students in the use of these skills, such practice enables the teacher to monitor student achievement in this area.

- Provide opportunities for students to "demonstrate" their understanding of concepts and skills studied through discussion, project work and group activities.

- "Observation" of the student in small group situations often provides insights as to the student's:

  - level of independence with the work
  - method of attacking problems
  - ability to apply concepts and skills to new situations.

- Structured interviews with students following completion of an assignment/project provides opportunity to evaluate student understanding of concepts/processes/applications. Such interviews may also suggest more effective ways of structuring future assignments.

- Evaluation should emphasize the "synthesis" of a variety of knowledge and process objectives, rather than isolated skills. Provide students with informal situations where they can demonstrate their application of basic number skills, algorithms, calculator skills, and estimation skills in solving a problem.

- Over-dependence on paper-and-pencil techniques often does not permit students with learning difficulties to do well. When planning formal evaluation procedures, caution should be exercised with regard to the overuse of:

  - multiple choice exams
  - difficult wording and vocabulary
  - simple recall of information without understanding and application.

- Provide encouragement by asking questions and making statements that will prompt students to evaluate their work and learning. Some examples might include:

  "You did a good job of ___(be specific)___."
  "What steps did you find most difficult?"
  "How could you improve your work in this question?"

  Such techniques will encourage students to be less "reward dependent" and more responsible for their own learning.

- Provide opportunities for the use of self and peer evaluation techniques.

- When planning formal evaluation (e.g., a quiz or a test), students should be given plenty of advance notice and a study guide to highlight the areas that need review. Students rarely do their best when caught off guard by "surprise" quizzes.

- Students often do not do their best under time pressure. Provide plenty of time for students to complete their work.

The *Teacher Resource Manual* contains additional guides and checklists for evaluating student performance and growth in mathematics.

SCOPE AND SEQUENCE

The scope and sequence chart provided on the following pages outlines the mathematical process and skill that is developmentally addressed throughout Mathematics 16 and 26. In recognizing that students differ in the rate at which they acquire mathematical competencies, the chart is intended to assist teachers in assessing present levels of student performance, and in diagnosing individual student strengths and weaknesses.

In using the scope and sequence, it should be noted that:

- although the scope and sequence describes the process and skill that is developmentally addressed throughout the two-year mathematics program, it does not provide a plan for instruction. **Instruction should be sequenced throughout Mathematics 16 and 26 in a manner that will suit individual student needs and growth patterns**

- learning objectives that provide a meaningful **context for instruction** (i.e., problem solving, use of technology, computational facility and estimation) are followed by learning objectives that support the development of process and skill within five **major strands** (i.e., number systems and operations, ratio, proportion and percent, algebra, geometry and measurement, data interpretation and display) established on the basis of tasks frequently demanded of students in everyday life

- effort has been made to arrange process and skill within each strand in a linear sequence according to cognitive demand at each grade level

- the skills are **developmental** through Mathematics 16 and 26 (i.e., the spiral approach). Mathematics 26 provides opportunities for students to reinforce and extend their understanding of skills developed in Mathematics 16 through more extensive applications, and also prescribes new skills related to each of the major content strands

- most students will continue to use concrete operational thinking, depending upon personal and tangible experience in order to link ideas. Instruction should generally **begin with an operational understanding of ideas** at the Mathematics 16 level, **and encourage students to extend their thinking to more abstract levels that are cognitively appropriate** at the Mathematics 26 level. (The *Teacher Resource Manuals* for Mathematics 16 and 26 provide for the development of thinking skills through themes that reflect increasing levels of analysis and critical thought)

- **the skills are interdependent and are not meant to be taught in isolation.** Although some skills may be mastered more effectively through discrete instruction, this approach is not advocated as a primary focus of instruction. The thematic structure permits a more holistic view of instruction through the linking of strategies and skills.

Teachers are encouraged to examine a scope and sequence chart for the I.O.P. Junior High School Mathematics Program (Grades 8 and 9). An understanding of the developmental progression of process and skill occurring in junior high school will facilitate program articulation and continuity.

CONTEXT FOR INSTRUCTION

PROBLEM SOLVING

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO USE A VARIETY OF STRATEGIES/SKILLS IN SOLVING PRACTICAL PROBLEMS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.	REFINES AND EXTENDS ABILITY TO USE A VARIETY OF STRATEGIES/SKILLS IN SOLVING INCREASINGLY DIFFICULT PROBLEMS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.
• Applies strategies/skills that are useful in understanding a problem. e.g., – reads the problem several times – asks questions and refers to other sources to clarify meaning – identifies key words and their meanings – looks for patterns – identifies given and needed information – identifies extraneous information – internalizes the problem by restating in his or her own words, or by visualizing the problem – draws pictures/diagrams – uses concrete manipulatives – interprets pictures/charts/graphs – relates the problem to other problems previously encountered – considers alternative interpretations of the problem.	• Applies strategies/skills that are useful in understanding a problem. e.g., – determines if there are hidden assumptions that contain information necessary to the solution of the problem.
• Applies strategies/skills that are useful in developing and carrying out a problem-solving plan. e.g., – guesses and checks the result (thus improving the guess) – uses logic or reason – identifies and applies relationships – chooses and sequences the operations needed – sorts and classifies information – applies selected strategies – presents ideas clearly – selects appropriate calculating/measuring devices and methods – acts out or simulates the problem – applies patterns – estimates the answer – documents the process used – works carefully – works in a group situation where ideas are shared – visualizes the problem – makes positive statements (e.g., "I can solve this.") – uses a simpler problem (making an analogy) – identifies factors relevant to the problem – collects and organizes data into diagrams, charts, tables, pictures, graphs or models – experiments through the use of manipulatives – breaks the problem down into smaller parts – formulates an equation – recognizes limits and eliminates possibilities.	• Applies strategies/skills that are useful in developing and carrying out a problem-solving plan. e.g., – constructs flow charts – works backward through the problem – examines the problem from varying perspectives/points of view
• Applies strategies/skills that are useful in reviewing and applying the results of problem solving. e.g., – states an answer to the problem – restates the problem with an answer – explains the answer in oral/written form – determines if the answer is reasonable – discusses with others the process used – suggests other ways of solving the problem – checks the answer – considers the possibility of other answers/solutions – makes and solves similar problems – generalizes the solution and applies the process used to other situations.	• Applies strategies/skills that are useful in reviewing and applying the results of problem solving. e.g., – creates other routine/non-routine problems that require the use of a similar process.

CONTEXT FOR INSTRUCTION (continued)

USE OF TECHNOLOGY

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO USE THE HAND-HELD CALCULATOR IN ROUTINE/NON-ROUTINE PROBLEM SITUATIONS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY. • Identifies appropriate and inappropriate uses of the calculator. • Clears the display of the calculator and corrects entry errors. • Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent: – identifies and uses basic functions (i.e., +, −, +, ×, %, =, decimal, clear) – enters numbers in correct sequence for subtraction and division – determines whole number remainders for division – uses subtotals and memory for adding, subtracting, multiplying and dividing – converts fractions to decimal and percent form – generates sets of multiples for a given number – follows order of operations. • Selects from calculator display the number of decimal places appropriate to the context of a calculation/problem. • Uses estimation and mental arithmetic to check the reasonableness of answers obtained on the calculator.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND PROFICIENCY WHEN USING THE HAND-HELD CALCULATOR IN ROUTINE/NON-ROUTINE PROBLEM SITUATIONS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY. • Identifies and uses additional functions on the calculator (i.e., $\times^2$, y^x, $\sqrt{\ }$).
DEMONSTRATES AN UNDERSTANDING OF THE BASIC OPERATION, CAPABILITIES, APPLICATIONS AND LIMITATIONS OF COMPUTERS. • Identifies major parts of a computer. • Distinguishes between hardware and software. • Explains how computers get their instructions from a program written to accomplish a specific task. • Gives examples of the applications of computers in performing tasks that require speed, accuracy, repeated operations and the processing of large amounts of data. • Identifies major areas in society where computers are used and the tasks performed by computers in these areas. • Gives examples of tasks that computers cannot accomplish.	DEMONSTRATES AN INCREASED LEVEL OF UNDERSTANDING OF THE OPERATION, CAPABILITIES, APPLICATIONS AND LIMITATIONS OF COMPUTERS.

CONTEXT FOR INSTRUCTION (continued)

COMPUTATIONAL FACILITY AND ESTIMATION

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO PERFORM COMPUTATIONS AS APPROPRIATE IN DAILY LIFE AND WORK-RELATED SITUATIONS, THROUGH THE USE OF MENTAL ARITHMETIC, PAPER-AND-PENCIL ALGORITHMS, THE CALCULATOR AND ESTIMATION.	DEMONSTRATES INCREASING LEVELS OF COMPETENCE AND ACCURACY WHEN PERFORMING COMPUTATIONS AS APPROPRIATE IN PRACTICAL SITUATIONS, THROUGH THE USE OF MENTAL ARITHMETIC, PAPER-AND-PENCIL ALGORITHMS, THE CALCULATOR AND ESTIMATION.
• Uses mental arithmetic skills that are based upon: – all single-digit operations – sequences of operations – doubling and halving – multiplying and dividing by powers of 10 – applications of the commutative, associative and distributive properties – properties of zero and one – strategies appropriate to the situation (e.g., compensation, computing from left to right).	→
• Uses paper-and-pencil algorithms to perform computations with whole numbers, decimals, fractions, integers and percent within parameters provided in the Scope and Sequence (see "Number Systems and Operations" and "Ratio, Proportion and Percent").	→
• Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent within parameters established by the nature of the problem (see "Use of Technology").	→
• Uses estimation skills that are based upon: – stating the largest and smallest reasonable answer to a problem before solving the problem – predicting whether a computation will result in a larger or smaller number – forecasting an order of magnitude for the result of a computation (e.g., 10's, 100's, 1000's) – predicting the magnitude of the result of a computation through use of strategies appropriate to the situation (e.g., front-end estimation, rounding, clustering, compatible numbers).	→

STRANDS

NUMBER SYSTEMS AND OPERATIONS

	MATHEMATICS 16	MATHEMATICS 26
WHOLE NUMBERS	APPLIES WHOLE NUMBER SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. • Identifies place value to one million. • Reads numbers to one million in context. • Write numbers in numeric or word form to one million. • Quantifies and orders numbers to one million. • Identifies even and odd numbers. • Rounds numbers to nearest 10, 100, 1000, 10 000 and 100 000. • Interprets/uses numerical terms (i.e., pair, triplet, decade, dozen, century). • Adds and subtracts whole numbers (limit for paper-and-pencil computation: numbers less than 10 000). • Uses mental arithmetic to determine products when multiplying by multiples of ten. • Multiplies whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100). • Applies divisibility rules for 2, 3, 5 and 10. • Divides whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100). • Determines the lowest common multiple for pairs of numbers less than 10. • Expresses numbers up to 50 as the product of prime factors. • Determines the greatest common factor for pairs of numbers less than 50.	REFINES AND EXTENDS ABILITY TO APPLY WHOLE NUMBER SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. → • Describes the meaning of terms related to the use of exponents (i.e., exponent, base, power, "squared", "cubed"). • Determines the value of a power, using whole number base and exponents of 2 and 3. • Determines the square root of whole numbers, using estimation/square root tables/calculator.
DECIMALS	APPLIES DECIMAL SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. • Identifies place value to thousandths. • Reads decimals to thousandths in context. • Writes decimals in numeric form to thousandths. • Quantifies and orders numbers to three decimal places in applications. • Rounds decimals to the nearest whole number, tenth and hundredth. • Adds and subtracts decimals (limit for paper-and-pencil computation: numbers to thousandths). • Multiplies decimals (limit for paper-and-pencil computation: numbers that yield products to thousandths, using one- or two-digit multipliers). • Divides decimals (limit for paper-and-pencil computation: numbers having five digits or less by one- or two-digit divisors; dividend may have 0, 1, 2 or 3 decimal places, divisor may be a whole number or decimal). • Uses mental arithmetic to determine products/quotients when multiplying/dividing by multiples of 10.	REFINES AND EXTENDS ABILITY TO APPLY DECIMAL SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS. → • Recognizes place value beyond the thousandths place in work-related situations (e.g., reading the micrometer). • Rounds to the nearest thousandth/ten thousandth as required in work-related situations.

NUMBER SYSTEMS AND OPERATIONS (cont'd)

	MATHEMATICS 16	MATHEMATICS 26
FRACTIONS	DEMONSTRATES ABILITY TO USE BASIC FRACTION SKILLS IN PRACTICAL PROBLEM SITUATIONS (WITH EMPHASIS ON FRACTIONS HAVING DENOMINATORS OF 2, 3, 4, 5, 8, 10 AND 16). • Illustrates a fraction as part of a whole, part of a set, or a point on a number line. • Describes proper/improper fractions and mixed numbers through the use of objects, pictures and diagrams. • Using a number line, illustrates the relationship between whole numbers, decimals and fractions. • Compares and orders fractions in applications. • Identifies and determines equivalent fractions (including proper and improper fractions, mixed numbers). • Identifies and expresses fractions in basic form. • Converts proper/improper/mixed fractions to decimals using a calculator. • Recalls decimal equivalents for one-half, thirds, quarters, fifths and tenths. • Determines common denominators for fractions having denominators of 2, 3, 4, 5, 8, 10 and 16. • Adds/subtracts fractions and mixed numbers having like/unlike denominators that are within the parameters stated above. • Multiplies/divides proper fractions and mixed numbers by whole numbers. • Calculates a fractional part of an amount in applications.	REFINES AND EXTENDS ABILITY TO USE BASIC FRACTION SKILLS IN PRACTICAL PROBLEM SITUATIONS (WITH EMPHASIS ON FRACTIONS HAVING DENOMINATORS OF 2, 3, 4, 5, 8, 10 AND 16). → • Multiplies/divides proper fractions and mixed numbers by whole numbers/proper fractions.
INTEGERS	DEMONSTRATES ABILITY TO USE INTEGER SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS. • Identifies situations that require the use of integers. • Uses vocabulary related to integers (i.e., positive, negative, plus, minus, above, below, gain, loss). • Places integers on the number line. • Compares and orders integers in practical applications. • Adds pairs/groups of one- and two-digit integers.	REFINES AND EXTENDS ABILITY TO USE INTEGER SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS. → • Adds/subtracts pairs of integers containing three digits or less. • Multiplies/divides pairs of one- and two-digit integers (where multiplier and divisor are positive). • Describes the use of integers in locating points on the coordinate plane.

STRANDS (continued)

RATIO, PROPORTION AND PERCENT

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO USE RATIO, PROPORTION AND PERCENT SKILLS IN EVERYDAY APPLICATIONS AND PROBLEM-SOLVING SITUATIONS.	REFINES AND EXTENDS ABILITY TO USE RATIO, PROPORTION AND PERCENT SKILLS IN EVERYDAY APPLICATIONS AND PROBLEM-SOLVING SITUATIONS
• Compares two quantities in the same unit by writing ratios in the form a/b, a:b and a to b.	→
• Generates equivalent ratios.	
• Calculates rates/unit rates by writing ratios that involve numbers with different units (e.g., 90 km/2h, 3 items for $1.00).	
• Identifies proportions as statements about equivalent ratios.	
• Writes proportions that describe practical problem situations.	
• Calculates the unknown value in a proportion.	
• Expresses whole number percents as ratios/decimals.	• Using a calculator, converts fractional percents to fraction and decimal forms (e.g., $12\ 1/2\% = \frac{1}{8} = 0.125$).
• Expresses ratios as decimals and percents (i.e., $\frac{a}{b} = \frac{?}{100}$ where b = 2, 4, 5, 10, 20, 25 or 50).	
• Recalls fraction, decimal and percent equivalents for one-half, quarters and tenths.	→
• Expresses one-, two- and three-place decimals as percents.	
• Determines other fraction, decimal and percent equivalents through the use of the calculator.	
• Calculates/estimates a percent of a number.	
	• Calculates/estimates what percent one number is of another.
	• Calculates/estimates any one of the missing elements in practical problems involving application of percent (i.e,. determines percentage, rate or base).

ALGEBRA

DEMONSTRATES ABILITY TO USE ALGEBRAIC SKILLS WITHIN THE CONTEXT OF EVERYDAY AND WORK-RELATED SITUATIONS.	DEMONSTRATES ABILITY TO USE ALGEBRAIC SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS THAT REQUIRE INCREASING LEVELS OF ANALYSIS AND CRITICAL THOUGHT.
• Distinguishes between the use of variables and constants in practical situations.	→
• Uses variables to write mathematical expressions/equations that describe familiar situations.	
• Evaluates expressions for given whole number values of the variable.	• Evaluates expressions for given whole number/decimal values of the variable.
• Uses estimation, guess/check, concrete manipulatives and formal strategies to find whole number solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$).	• Uses estimation, guess/check, concrete manipulatives and formal strategies to find whole number/decimal solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$).
• Applies equation-solving skills to practical problem situations.	• Applies equation-solving strategies/skills to practical situations involving whole numbers/decimals.
• Interprets formulas as word statements.	
• Uses formulas and substitution to solve problems in practical situations (e.g., if $P = 2L + 2w$, find P when L = 10 and w = 8).	• Uses substitution and equation-solving strategies to find the missing element in a formula (e.g., if $P = 2L + 2w$, find L when P = 16 and w = 3).

GEOMETRY AND MEASUREMENT

GEOMETRY

MATHEMATICS 16	MATHEMATICS 26
DEMONSTRATES ABILITY TO APPLY GEOMETRY SKILLS IN DAILY LIFE AND WORK-RELATED SITUATIONS.	DEMONSTRATES INCREASING CONFIDENCE AND ABILITY TO APPLY GEOMETRY SKILLS IN DAILY LIFE AND WORK-RELATED SITUATIONS.
• Identifies line relationships (i.e., horizontal, vertical, parallel, perpendicular and intersecting).	→
• Identifies/constructs basic two-dimensional geometric figures (i.e., rectangle, square, parallelogram, triangle).	
• Classifies triangles and quadrilaterals by examining measures of sides and measures of angles.	• Identifies/classifies polygons according to the number of sides.
• Identifies the relationship between the radius and diameter of a circle.	• Constructs regular polygons (i.e., triangle, square, hexagon, octagon).
• Constructs circles, given either radius or diameter.	
• Identifies similar and congruent geometric figures.	→
• Identifies/sketches/constructs models of basic three-dimensional shapes (i.e., rectangular solid, cube, cylinder, cone).	
• Constructs geometric figures/designs, using a variety of construction tools (i.e., straightedge, ruler, compass, protractor, computer).	• Describes the terminology associated with the coordinate system (i.e., origin, axis, quadrant, coordinates, ordered pair).
	• Plots a point on the coordinate plane when given its coordinates, and identifies the coordinates for a point graphed in the coordinate plane.

LENGTH

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE LENGTH IN DAILY LIFE AND WORK-RELATED SITUATIONS.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING LENGTH IN DAILY LIFE AND WORK-RELATED SITUATIONS.
• Identifies common SI units of length (i.e., mm, cm, m, km).	→
• Estimates/measures length, using metric units and tools appropriate to the situation.	
• Converts among SI units of length as required in applications.	
• Estimates/measures/calculates the perimeter of figures bounded by line segments using units and tools appropriate to the situation.	
• Identifies the value of π as the ratio of the circumference of a circle to its diameter.	• Develops/applies strategies for determining the perimeter of any regular polygon.
• Develops/applies strategies for determining the circumference of a circle.	• Describes the Pythagorean relationship among sides of a right triangle, and applies the relationship to practical situations.

AREA

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE AREA IN DAILY LIFE AND WORK-RELATED SITUATIONS.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING AREA IN DAILY LIFE AND WORK-RELATED SITUATIONS.
• Identifies common SI units of area (i.e., cm^2, m^2) and their application in practical situations.	→
• Approximates the area of two-dimensional geometric figures using a square grid.	
• Estimates/calculates area of the rectangle, square, triangle and circle, using units, tools and strategies appropriate to the situation.	• Develops/applies appropriate strategies for determining the surface area of rectangular solids and cylinders.

STRANDS (continued)

GEOMETRY AND MEASUREMENT (cont'd)

	MATHEMATICS 16	MATHEMATICS 26
VOLUME	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE VOLUME IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of volume (i.e., cm^3, m^3) and their application in practical situations. • Approximates the volume of rectangular solids using cubes. • Estimates/calculates the volume of rectangular solids and cubes, using units, tools and strategies appropriate to the situation.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING VOLUME IN DAILY LIFE AND WORK-RELATED SITUATIONS. → • Identifies the equivalence between cubic centimetres and millilitres. • Develops/applies appropriate strategies for determining the volume of right cylinders.
CAPACITY	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE CAPACITY IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of capacity (i.e., mL, L) and their application in practical situations. • Estimates and measures capacity, using metric units and tools appropriate to the situation. • Converts between mL and L.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING CAPACITY IN DAILY LIFE AND WORK-RELATED SITUATIONS. →
MASS	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE MASS IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies common SI units of mass (i.e., g, kg, t) and their application in practical situations. • Estimates and measures mass, using metric units and tools appropriate to the situation. • Converts between g and kg and between kg and t.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING MASS IN DAILY LIFE AND WORK-RELATED SITUATIONS. →
TIME	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE TIME IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Uses a calendar, recognizing the relationship between days, weeks, months and years. • Uses National Standards for numeric dating. • Estimates/measures/records time on the 12-hour and 24-hour clocks, using traditional and digital time pieces. • Determines the interval between a pair of times. • Determines the finishing time, given the starting time and duration.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILLS WHEN ESTIMATING AND MEASURING TIME IN DAILY LIFE AND WORK-RELATED SITUATIONS. →
TEMPERATURE		DEMONSTRATES ABILITY TO ESTIMATE AND MEASURE TEMPERATURE IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Estimates/measures temperature on the Celsius scale. • Recalls important temperatures on the Celsius scale (i.e., boiling/freezing point of water, normal room/body temperature). • Determines temperature change, including changes from below zero to above zero and vice versa.
ANGLE	DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE ANGLES IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Identifies an angle and the degree as a unit of measure. • Identifies/estimates angles of 30°, 45°, 60°, 90°, 180° and 360°. • Uses a protractor to measure and draw angles from 0° to 180°.	DEMONSTRATES INCREASING LEVELS OF CONFIDENCE AND SKILL WHEN ESTIMATING AND MEASURING ANGLES IN DAILY LIFE AND WORK-RELATED SITUATIONS. • Uses a protractor to measure and draw angles from 0° to 360°. • Applies knowledge of the sum of the angles of a triangle in practical situations.

DATA INTERPRETATION AND DISPLAY

MATHEMATICS 16	MATHEMATICS 26
COLLECTS, ORGANIZES, DISPLAYS AND ANALYZES NUMERICAL DATA IN ORDER TO DRAW VALID INFERENCES AND MAKE INFORMED DECISIONS IN PRACTICAL EVERYDAY SITUATIONS. • Distinguishes between a survey and census, also between a sample and population, identifying potential biases that may occur in surveys and samples. • Interprets and analyzes data presented in tables/charts and bar/line/picture/circle graphs. • Displays data in the form of tables/charts and bar/line/picture/circle graphs. • Identifies the characteristics of a misleading graphical display. • Determines/calculates appropriate measures of central tendency in practical situations (i.e., mean, median, mode, range). • Conducts a survey or poll, using appropriate methods of gathering, organizing, presenting and analyzing data: – selects a suitable sample from a given population – collects and organizes data using tally sheets/frequency tables – displays data in the form of bar/line/picture/circle graphs – makes inferences/generalizations about the population from which a sample was taken.	COLLECTS, ORGANIZES, DISPLAYS AND ANALYZES NUMERICAL DATA IN ORDER TO DRAW VALID INFERENCES AND MAKE INFORMED DECISIONS IN PRACTICAL SITUATIONS THAT REQUIRE INCREASING LEVELS OF ANALYSIS AND CRITICAL THOUGHT. → • Explains the meaning of "probability" and gives examples of its use in today's society. • Uses counting techniques such as tree diagrams to find the number of possible outcomes of an experiment. • Expresses the probability of the occurrence of a single event related to practical experience or a simple experiment.

PROGRAM OF STUDIES/PRESENTATION OF CONTENT

The "Program of Studies/Presentation of Content" describes the required concepts, skills and attitudes identified for Mathematics 16. Learning objectives that provide the context for instruction are followed by learning objectives that correspond to each of the five strands established for the program, and include:

- Problem Solving
- Use of Technology
- Computational Facility and Estimation

context for instruction

- Number Systems and Operations
- Ratio, Proportion and Percent
- Geometry and Measurement
- Data Interpretation and Display
- Algebra.

strands

The shaded statements contained in the Program of Studies (Column One) enable the reader to readily identify those portions of the program that are prescribed.

The advice and direction offered throughout Columns Two, Three and Four is not prescriptive, and is offered only as a service to teachers. Column Two, Related Life Skills, establishes an immediate need or use for each skill being studied and suggests ways of planning relevant learning experiences. Further suggestions for relating mathematical competencies to applications in occupational courses and other academic disciplines are provided in Column Three, Related Applications Across the Curriculum. The references provided in this column will facilitate curricular integration by establishing a base for cooperative planning among other subject areas/teachers in the program. A variety of strategies useful in addressing developmental characteristics and learning styles of the student are provided in Column Four, Suggested Strategies/Activities. Teachers are encouraged to use the advice and direction offered throughout Columns Two, Three and Four in conjunction with suggestions provided in the *Teacher Resource Manual* when planning thematic units of instruction that address learning objectives identified in the Program of Studies.

MATHEMATICS 16

PROBLEM SOLVING

Problem solving is to be integrated throughout all areas of the mathematics program, as learning to solve problems is a major purpose of studying mathematics. Students should recognize that problems can be solved in many ways, and will be expected to use a variety of strategies that help them to understand problems, develop and carry out a problem-solving plan, and review/apply the results of problem solving.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- problems can be routine, requiring only the application of a known procedure/algorithm, or non-routine, requiring the development of a process or the conducting of an investigation
- a variety of strategies/skills are useful in understanding and solving mathematical problems
- computational facility, involving the use of paper-and-pencil algorithms, estimation, mental arithmetic and/or the calculator may assist the problem-solving process.

SKILLS

The skills relating to **PROBLEM SOLVING** are outlined in the Learning Objectives listed in Column One on the pages that follow.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward the application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- appreciate the value of an organized approach to problem solving
- display an attitude of curiosity and openness to new ideas, and to be critical and constructive when selecting and applying problem-solving strategies in practical situations
- appreciates the usefulness of problem-solving skills in consumer and work-related situations.

LEARNING OBJECTIVES	Related Life Skills

DEMONSTRATES ABILITY TO USE A VARIETY OF STRATEGIES/SKILLS IN SOLVING PRACTICAL PROBLEMS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.

- Applies strategies/skills that are useful in understanding a problem.
  e.g., – reads the problem several times
  – asks questions and refers to other sources to clarify meaning
  – identifies key words and their meanings
  – looks for patterns
  – identifies given and needed information
  – identifies extraneous information
  – internalizes the problem by restating in his or her own words, or by visualizing the problem
  – draws pictures/diagrams
  – uses concrete manipulatives
  – interprets pictures/charts/graphs
  – relates the problem to other problems previously encountered
  – simulates/models the problem situation
  – considers alternative interpretations of the problem.

- Applies strategies/skills that are useful in developing and carrying out a problem-solving plan.
  e.g., – guesses and checks the result (thus improving the guess)
  – uses logic or reason
  – identifies and applies relationships
  – chooses and sequences the operations needed
  – sorts and classifies information
  – applies selected strategies
  – presents ideas clearly
  – selects appropriate calculating/measuring devices and methods
  – acts out or simulates the problem
  – applies patterns
  – estimates the answer
  – documents the process used
  – works carefully
  – works in a group situation where ideas are shared
  – visualizes the problem
  – makes positive statements (e.g., "I can solve this.")
  – uses a simpler problem (making an analogy)
  – identifies factors relevant to the problem.

Recognizes that computational competence may assist the problem-solving process. Performs computations using whole numbers/decimals/fractions/integers when solving real life problems related to:

- personal finance/management
- home maintenance
- travel/sport/recreation.

Recognizes that in solving problems, computation may involve the use of:

- paper-and-pencil algorithms
- estimation
- mental arithmetic
- the calculator.

Applies appropriate problem-solving strategies to situations that are routine, involving the application of specific concepts/skills:

- determines the quantity/cost of materials required for a project
- determines the "best buy" by comparing the unit price of competing products.

Applies appropriate problem-solving strategies to situations that are non-routine and open-ended, requiring the use of creative thinking and ability to recognize different solutions:

- selects the most appropriate store for purchase of sportswear after considering selection/quality/cost/service at a number of competing stores
- examines alternative sources of consumer loans and chooses the best alternative after considering interest rates/duration of loan/collateral needed/non-payment penalties.

Related Applications Across the Curriculum

General

The use of appropriate strategies for problem solving is emphasized throughout all subject areas. A problem-solving model similar to the model used in mathematics is frequently applied to real life situations in:

- English
- science
- social studies
- occupational courses.

English

Identifies problem-solving/decision-making strategies used by characters in literature. Develops and assesses alternative strategies.

Science

Uses appropriate thought processes and thinking strategies when:

- conducting scientific inquiry
- solving technological problems
- making responsible decisions in society.

Social Studies

Uses number systems/operations when reading and interpreting graphs/charts/surveys/polls.

Uses problem-solving and decision-making strategies to investigate:

- discrimination and prejudice
- rights and responsibilities of individuals and society.

Occupational Courses

Uses problem-solving and decision-making strategies to:

- design/produce a product
- provide a service.

Suggested Strategies/Activities

Problem solving must be integrated throughout all areas of the mathematics program, and should be developed within the context of real life and work-related situations. The learning objectives outlined on the previous page provide an overview of the strategies/skills that should be developed through themes provided in the *Teacher Resource Manual*.

While instructional activities will continue to include the use of routine problems that require the application of familiar techniques, emphasis should be placed on the use of non-routine and open-ended problem situations. Students must be given opportunities to exercise creative/divergent thinking skills and to recognize different solutions in the problems they experience. Individual differences must be considered in providing students with problems that are appropriate to their level of ability.

Encourage students to accept and appreciate that being perplexed and unsure is often normal when first encountering a problem situation. Create a positive classroom atmosphere that allows students to foster their own ideas and approaches to problem solving. Student attitudes and beliefs that build self-confidence in problem situations include:

- "Problems can be solved in more than one way"
- "Problems may have more than one answer"
- "If the first strategy I try doesn't work, I'll try to find another strategy".

Model the problem-solving process for students. Facilitate active involvement in problem solving by encouraging students to:

- ask questions
- manipulate materials
- discuss and share ideas/strategies in small groups
- verbalize their ideas and discoveries
- think of related problems from personal experience.

LEARNING OBJECTIVES	Related Life Skills
• Applies strategies/skills that are useful in developing and carrying out a problem-solving plan (continued). e.g., – collects and organizes data into diagrams, charts, tables, pictures, graphs or models – experiments through the use of manipulatives – breaks the problem down into smaller parts – formulates an equation – recognizes limits and eliminates possibilities.	Distinguishes between real life problems that require exact answers and those that require only an estimation. Selects methods of computation that are appropriate to each type of problem-solving situation. Develops a habit of using estimation to check the reasonableness of computational results obtained using paper and pencil or the calculator: • "Is my answer reasonable?" • "Within what range of numbers must my answer lie?"
• Applies strategies/skills that are useful in reviewing and applying the results of problem solving. e.g., – states an answer to the problem – restates the problem with an answer – explains the answer in oral/written form – determines if the answer is reasonable – discusses with others the process used – suggests other ways of solving the problem – checks the answer – considers the possibility of other answers/ solutions – makes and solves similar problems – generalizes the solution and applies the process used to other situations.	

Related Applications Across the Curriculum	Suggested Strategies/Activities
	Provide opportunities for students to increase their proficiency in the use of all methods of computation (i.e., mental arithmetic, paper-and-pencil, calculator, estimation). Coach students in the discerning use of each method in the problem-solving process, thus avoiding singular emphasis on the use of paper-and-pencil algorithms. Students should not be evaluated on their ability to solve problems simply by what they produce on paper. Attitudes and behaviours also need to be taken into account. Evaluation procedures should include a variety of techniques, including: • observation • interviews • inventories/checklists • anecdotal records • written assignments. Appropriate evaluation techniques for problem solving are further discussed and illustrated in the *Teacher Resource Manual* (see "Problem Solving" and "Assessment/Evaluation").

MATHEMATICS 16

USE OF TECHNOLOGY

The mathematics program must recognize the pervasiveness of technology by de-emphasizing activities that are much more easily replicated by calculators and computers. Each student should have access to a hand-held calculator, and will be expected to become proficient and discerning in its use. To the extent that computer facilities and equipment are available, students should be given opportunity to work independently with prepared software and to use simple programs that have been written for particular purposes.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- calculators and computers have influenced the nature of the computational procedures and problem-solving processes that we use
- computer technologies have basic capabilities and limitations that determine the nature of the tasks they perform
- effective use of calculators and computers requires knowledge of appropriate procedures for their use.

SKILLS

The skills relating to **USE OF TECHNOLOGY** are outlined in the Learning Objectives listed in Column One on the following page.

ATTITUDES

Students are encouraged to:

- display a positive attitude toward the application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- demonstrate confidence in ability to use a calculator effectively in problem-solving situations that require quantitative thinking and computational facility
- appreciate current and potential impacts of computer technology in everyday and work-related situations.

COURSE: MATHEMATICS 16
USE OF TECHNOLOGY

LEARNING OBJECTIVES	Related Life Skills

DEMONSTRATES ABILITY TO USE THE HAND-HELD CALCULATOR IN ROUTINE/NON-ROUTINE PROBLEM SITUATIONS THAT REQUIRE QUANTITATIVE THINKING AND COMPUTATIONAL FACILITY.

- Identifies appropriate and inappropriate uses of the calculator.
- Clears the display of the calculator and corrects entry errors.
- Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent:
  - identifies and uses basic functions (i.e., +, −, ×, ÷, %, =, decimal, clear)
  - enters numbers in correct sequence for subtraction and division
  - determines whole number remainders for division
  - uses subtotals and memory for adding, subtracting, multiplying and dividing
  - converts fractions to decimal and percent form
  - generates sets of multiples for a given number
  - follows order of operations.
- Selects from calculator display the number of decimal places appropriate to the context of a calculation/problem.
- Uses estimation and mental arithmetic to check the reasonableness of answers obtained on the calculator.

DEMONSTRATES AN UNDERSTANDING OF THE BASIC OPERATION, CAPABILITIES, APPLICATIONS AND LIMITATIONS OF COMPUTERS.

- Identifies major parts of a computer.
- Distinguishes between hardware and software.
- Explains how computers get their instructions from a program written to accomplish a specific task.
- Gives examples of the applications of computers in performing tasks that require speed, accuracy, repeated operations and the processing of large amounts of data.
- Identifies major areas in society where computers are used and the tasks performed by computers in these areas.
- Gives examples of tasks that computers cannot accomplish.

Follows appropriate procedures for storing, handling and operating calculators and computer hardware/software.

Uses a calculator in situations where mental arithmetic would be difficult, or when paper-and-pencil computation would be inefficient:

- determining quantity/cost of materials required for a construction/repair project
- monitoring total cost of a number of items being selected for purchase
- determining unit price and "best buy"
- verifying sales slips/invoices/bills
- maintaining bank account balances
- determining gross pay and net pay
- preparing an income tax return
- comparing loan costs
- monitoring fixed and variable costs in operating a business/home.

Makes a habit of using estimation to check the reasonableness of computational results obtained using the calculator.

Uses system commands to run prepared programs on a computer. Locates/uses alphanumeric and special function keys on a computer keyboard.

Recognizes the capabilities of computers in preparing graphical representations.

Understands the applications of computer spreadsheets.

Identifies the tasks performed by computer technologies in a variety of real life situations:

- banking
- consumer transactions.

Appreciates current and potential impacts of computer technology on:

- employment and career opportunities
- career education and job training.

Related Applications Across the Curriculum

English

Uses appropriate computer technologies as an aid to:

- conducting research and gaining access to information
- reporting/communicating information and ideas.

Science

Uses a calculator when performing calculations in a variety of classroom/laboratory situations.

Identifies various tasks performed by computers in:

- managing environment and resources
- monitoring personal health.

Recognizes the impact of computer technologies on personal lifestyle and employment/career opportunities.

Social Studies

Uses a calculator when:

- determining distances on a map
- interpreting statistical data relating to laws, government and employment.

Recognizes the impact of computer technologies on:

- employment/career opportunities
- crime detection/law enforcement
- medicine and health care
- voting procedures.

Occupational Courses

Uses a calculator to perform computations within the context of managing finances, inventory and other quantifiable materials.

Recognizes the impact of computer technology on:

- the manufacturing industry
- business practices/procedures
- employment/career opportunities.

Suggested Strategies/Activities

Calculators and computer technology should be used by students throughout the mathematics program in support of instruction, class work, assignments and evaluation. Their use will decrease the time spent on tedious computation, thus allowing for an increase in time and emphasis on cognitive process and problem solving.

Students should use calculators on a regular basis when the purpose of an activity is the development of problem solving or other skill in which computation is of secondary importance. Be prepared to provide instruction on how to use the calculator in problem-solving contexts. Activities with the calculator should focus attention on:

- knowledge and use of place value, basic number skills and arithmetical operations
- proper documentation of numbers and operations that are used
- the development of skills in estimation and mental arithmetic
- ability to judge the reasonableness of an answer
- an appreciation of the importance of making a judgment about the results of computation.

For most students, the calculator should be used after mathematical concepts and algorithms are understood. Students who simply cannot master their basic number skills or who have persistent problems with computation should use calculators more extensively.

Provide opportunities for students to work with prepared software on a computer, and to enter, run and modify simple "user friendly" computer programs. These experiences will support concept and skill development, while at the same time enable students to interact with computers and gain first-hand knowledge of their versatility and limitations. Select computer programs and software on the basis of their effectiveness in:

- developing mathematical concepts
- developing problem-solving skills
- providing for drill and practice.

The *Teacher Resource Manual* provides additional suggestions on how calculators and computer technology can be used in regular classroom instruction throughout the mathematics program (see "Themes" and "Use of Technology").

MATHEMATICS 16

COMPUTATIONAL FACILITY AND ESTIMATION

Students must recognize that there are several ways to perform numerical computations, and that the method chosen will depend upon the situation at hand. Students will be expected to develop strategies for the use of all methods of computation (i.e., mental arithmetic, paper and pencil, calculator, estimation), and recognize when each method is most appropriately used in everyday problem-solving situations. It is intended that related concepts, skills and attitudes be developed/reinforced throughout the mathematics program, within the context of real life and work-related situations.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- computation can be performed by a variety of methods, and may include the use of mental arithmetic, paper-and-pencil algorithms, estimation and the calculator
- one must consider the degree of precision/accuracy that is required when selecting a method of computation that is appropriate to a particular problem situation
- checking an answer for its reasonableness and accuracy is a fundamental step in the computational process
- estimation is useful in determining the reasonableness of the results of computation and problem solving.

SKILLS

The skills relating to **COMPUTATIONAL FACILITY AND ESTIMATION** are outlined in the Learning Objectives listed in Column One on the following page.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward the application of concepts and skills of mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- demonstrate flexibility and ingenuity when selecting and applying appropriate methods of computation in daily life and work-related situations
- appreciate the usefulness of computational facility and estimation in solving everyday problems and making informed decisions.

COURSE: MATHEMATICS 16
COMPUTATIONAL FACILITY AND ESTIMATION

LEARNING OBJECTIVES

DEMONSTRATES ABILITY TO PERFORM COMPUTATIONS AS APPROPRIATE IN DAILY LIFE AND WORK-RELATED SITUATIONS, THROUGH THE USE OF MENTAL ARITHMETIC, PAPER-AND-PENCIL ALGORITHMS, THE CALCULATOR AND ESTIMATION.

- Uses mental arithmetic skills that are based upon:
  - all single-digit operations
  - sequences of operations
  - doubling and halving
  - multiplying and dividing by powers of 10
  - applications of the commutative, associative and distributive properties
  - properties of zero and one
  - strategies appropriate to the situation (e.g., compensation, computing from left to right).

- Uses paper-and-pencil algorithms to perform computations with whole numbers, decimals, fractions, integers and percent within parameters provided in the Scope and Sequence (see "Number Systems and Operations" and "Ratio, Proportion and Percent").

- Uses the calculator to perform computations with whole numbers, decimals, fractions, integers and percent within parameters established by the nature of the problem (see "Use of Technology").

- Uses estimation skills that are based upon:
  - stating the largest and smallest reasonable answer to a problem before solving the problem
  - predicting whether a computation will result in a larger or smaller number
  - forecasting an order of magnitude for the result of a computation (e.g., 10's, 100's, 1000's)
  - predicting the magnitude of the result of a computation through use of strategies appropriate to the situation (e.g., front-end estimation, rounding, clustering, compatible numbers).

Related Life Skills

Distinguishes between real life situations that require exact answers and those that require only an estimation. Selects a method of computation appropriate to each situation:

- "Do I have enough money to buy these articles?"
- "Brand X is the best buy."
- "A 15% tip would be about . . . dollars."
- "My net pay will be . . . ."

Uses mental arithmetic in daily activities:

- "How much will I pay for both the tape and the shirt?"
- "Have I received the correct amount of change?"
- "How much sugar will I use if I double the recipe?"

Uses a calculator in situations where mental arithmetic would be difficult, or when paper-and-pencil computation would be inefficient.

Makes a habit of using estimation to check the reasonableness of computational results obtained using paper and pencil or the calculator:

- "Is my answer reasonable?"
- "Within what range of numbers must my answer lie?"

Uses estimation skills in daily life situations where approximate numbers or answers are more appropriate than exact numbers:

- "Is there enough gas in the car?"
- "How much should I tip?"
- "How many people were at the game?"
- "How long will the trip take?"
- "Is this a reasonable price to pay?"
- "Is the answer on my calculator 'in the ballpark'?"

Applies appropriate strategies of estimation in consumer-related situations:

- selects items for purchase on the basis of approximate cost comparisons
- approximates total cost of items selected for purchase
- checks accuracy of sales receipts
- anticipates change due when making a purchase.

Related Applications Across the Curriculum	Suggested Strategies/Activities
General Computational procedures are used to varying degrees throughout all subject areas. Ensuing pages of this document will describe: • the context in which computation is used in other subject areas • the nature of the skills used by students.	Provide opportunities throughout the mathematics program for students to increase their proficiency in the use of all methods of computation (i.e., mental arithmetic, paper-and-pencil, calculator, estimation). Coach students in the discerning use of each method in the problem-solving process, thus avoiding singular emphasis on the use of paper-and-pencil algorithms. Maintain and develop mental arithmetic skills on a regular basis through timed challenges and games that are appropriate to student ability (see *Teacher Resource Manual*: Computational Facility and Estimation). Avoid embarrassing the student who experiences difficulty with these challenges by emphasizing self-competition and improvement, rather than team competition. Model "easy to use" strategies that will assist students to compute mental exact answers in situations of increasing difficulty. Encourage students to share strategies they find useful with other members of the class. Instruction in paper-and-pencil computation should continue to emphasize understanding of process (i.e., place value, regrouping, borrowing) and why algorithms are constructed in particular forms. Reinforce these understandings through work with one-, two- and three-digit numbers. Tedious paper-and-pencil computations with numbers containing more than four digits is discouraged. Such computations are time-consuming, and are generally ineffective in facilitating the development of number sense and problem-solving ability. Estimation involves application of a set of skills that are developed over time through much opportunity for practice. Appropriate strategies for the development of estimation skills are provided in the *Teacher Resource Manual*. Model these strategies and "think out loud" with students. Encourage the use of estimation skills on a daily basis in activities involving: • applications • problem solving • the use of calculators. Estimation skills are best evaluated through daily questioning and informal observation. Formal assessment of these skills can be counter-productive, as it may encourage students to make an approximation after obtaining an exact answer.
Science Selects/uses computational methods (i.e., mental arithmetic, paper and pencil, calculator, estimation) that are appropriate to a variety of classroom/laboratory situations.	
Social Studies Selects/uses methods of computation that are appropriate to the situation when: • working with distances and scales on a map • interpreting/analyzing statistical data related to various topics. Uses estimating and predicting skills to hypothesize the outcome of a current news issue.	
Occupational Courses Performs computations within the context of managing: • finances • inventory • other quantifiable materials. Selects a method of computation (i.e., mental arithmetic, paper and pencil, calculator, estimation) that is appropriate to the nature of the problem or purpose of the activity.	

MATHEMATICS 16

NUMBER SYSTEMS AND OPERATIONS

Students will be expected to demonstrate knowledge of the basic properties of whole numbers, decimals, fractions and integers, and of appropriate strategies for performing operations with these numbers. It is intended that related concepts and skills be developed/reinforced within the context of real life and work-related situations, rather than as a topic on their own.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- effective use of whole numbers, decimals, fractions and integers requires knowledge of appropriate notation and number properties
- performing operations of addition, subtraction, multiplication and division with whole numbers, decimals, fractions and integers involves the application of certain processes
- checking an answer for reasonableness and accuracy is a fundamental step in performing operations with number systems
- computation with whole numbers, decimals, fractions and integers may involve the use of paper-and-pencil algorithms, estimation, mental arithmetic and/or the calculator.

SKILLS

The skills relating to **NUMBER SYSTEMS AND OPERATIONS** are outlined in the Learning Objectives listed in Column One on the pages that follow.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward the application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- appreciate the usefulness of whole numbers, decimals, fractions and integers in practical everyday activities and work-related situations
- display an attitude of curiosity and openness to new ideas, and to be critical and constructive when selecting and applying computational procedures in problem-solving situations.

LEARNING OBJECTIVES	Related Life Skills

WHOLE NUMBERS

APPLIES WHOLE NUMBER SKILLS TO PRACTICAL PROBLEM SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS.

- Identifies place value to one million.
- Reads numbers to one million in context.
- Writes numbers in numeric or word form to one million.
- Quantifies and orders numbers to one million.
- Identifies even and odd numbers.
- Rounds numbers to nearest 10, 100, 1 000, 10 000 and 100 000.
- Interprets/uses numerical terms (i.e., pair, triplet, decade, dozen, century).
- Adds and subtracts whole numbers (limit for paper-and-pencil computation: numbers less than 10 000).
- Uses mental arithmetic to determine products when multiplying by multiples of ten.
- Multiplies whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100).
- Applies divisibility rules for 2, 3, 5 and 10.
- Divides whole numbers (limit for paper-and-pencil computation: numbers less than 10 000 by numbers less than 100).
- Determines the lowest common multiple for pairs of numbers less than 10.
- Expresses numbers up to 50 as the product of prime factors.
- Determines the greatest common factor for pairs of numbers less than 50.

Reads and interprets whole number calibrations on familiar meters, gauges and tools:

- kitchen/workshop measuring tools
- odometer
- air pressure gauge
- gas pump meter
- natural gas/electric/water meter.

Sequences and ranks whole numbers when:

- interpreting/displaying statistical data
- locating addresses/stadium seats
- locating information in the library.

Interprets numerical patterns/codes that are used in:

- house numbers/room numbers
- lock combinations
- highway routes/bus routes
- postal codes/telephone numbers.

Counts by multiples when quantifying:

- time on a traditional timepiece
- value of currency (coins or bills)
- chairs in the gym/cars in the parking lot
- total inventory on hand.

Uses the correct order for operations in answering skill-testing questions used in contests and promotions.

Reads and interprets distance charts and highway maps, calculating distances between cities.

Applies appropriate computational strategies (i.e., mental arithmetic, paper and pencil, calculator, estimation) in determining:

- materials required for construction and repair projects undertaken at home (e.g, number of tiles/bricks/panels/litres of paint/metres of trim)
- travel time and fuel consumption for given travel distances and travel rates.

Related Applications Across the Curriculum

English

Uses comprehension skills in reading/understanding problems that involve whole number/decimal operations:

- applies word recognition skills
- observes sequence of events
- identifies main idea and supporting detail
- uses visual/graphic information
- recognizes non-relevant detail.

Science

Uses knowledge of whole numbers and place value when:

- reading/interpreting calibrations on meters/gauges/scales
- gathering/interpreting statistical data
- using scientific formulas (e.g., mechanical advantage, heart beat recovery rate)
- performing computations using paper and pencil, mental arithmetic, estimation and the calculator.

Social Studies

Uses whole numbers when:

- relating an address to a city/town grid system
- comparing the remuneration offered for different types of work
- discussing laws and bylaws (e.g., driving age, speed limits)
- comparing product warranties.

Occupational Courses

Uses whole number computational skills in solving problems that involve:

- counting, calculating and recording movement of inventory
- reading and interpreting numbers on gauges/meters/scales.

Suggested Strategies/Activities

Develop whole number concepts within the context of real life and work-related situations. The learning objectives outlined on the preceding page provide an overview of whole number skills that should be developed/reinforced through the delivery of themes suggested in the *Teacher Resource Manual*.

Encourage students to discover whole number patterns and relationships throughout each theme by:

- exploring/experimenting
- simulating problem situations
- using the calculator
- working in small groups
- discussing/sharing ideas.

Provide frequent opportunities for the application of basic number skills. While most students may be able to recall the sums, differences, products and quotients for single digit numbers, individuals who continue to experience difficulty with these will need to employ other strategies (e.g., calculator, tables, repeated additions/subtractions, finger math) in order to develop an understanding of process and consolidate needed computational skills.

Be aware that students often experience difficulty in understanding the abstract processes involved in:

- rounding beyond 10's and 100's (e.g., 1000's, 10 000's, 100 000's)
- three-digit and four-digit subtraction that involves regrouping over a zero digit
- multiplication/division of numbers by 10 and multiples of 10
- multiplication when the multiplier contains zeroes
- division containing zeroes in the quotient
- division by two-digit divisors.

Strategies that are effective in developing these concepts must be identified and frequently modelled as related skills are used in a variety of contexts. Even at the high school level, manipulative materials must be used to enable students to internalize concepts (see *Teacher Resource Manual*: Situational and Concrete Approaches).

Avoid evaluation techniques that over-emphasize the use of paper-and-pencil algorithms. While these skills are important, evaluation should also assess student ability to:

- effectively use the calculator
- compute with mental arithmetic
- estimate computational results
- use concepts/skills in applications and problem solving.

LEARNING OBJECTIVES	Related Life Skills

DECIMALS

APPLIES DECIMAL SKILLS TO PRACTICAL PROBLEM-SOLVING SITUATIONS INVOLVING SEVERAL STEPS AND OPERATIONS.

- Identifies place value to thousandths.
- Reads decimals to thousandths in context.
- Writes decimals in numeric form to thousandths.
- Quantifies and orders numbers to three decimal places in applications.
- Rounds decimals to the nearest whole number, tenth and hundredth.
- Adds and subtracts decimals (limit for paper-and-pencil computation: numbers to thousandths).
- Multiplies decimals (limit for paper-and-pencil computation: numbers that yield products to thousandths, using one- or two-digit multipliers).
- Divides decimals (limit for paper-and-pencil computation: numbers having five digits or less by one- or two-digit divisors; dividend may have 0, 1, 2 or 3 decimal places, divisor may be a whole number or decimal).
- Uses mental arithmetic to determine products/quotients when multiplying/dividing by multiples of 10.

Reads meters, gauges and scales found in the home that are calibrated in tenths, hundredths and thousandths.

Establishes a practice of rounding calculations that represent money to the nearest hundredth (i.e., nearest cent).

Recognizes multi-step problems that may require rounding to the nearest tenth (or hundredth) in order to maintain a degree of accuracy.

Multiplies/divides by multiples of ten in converting metric units of measure.

Applies appropriate computational strategies (i.e., mental arithmetic, paper and pencil, calculator, estimation) in consumer-related situations:

- determines unit price/multiple price
- determines "best buy" by comparing the unit price of competing brands
- determines discount and sale price
- maintains a running total of the cost of items being selected for purchase
- determines the total cost of items purchased
- determines change due on purchases made
- checks the accuracy of sales-slip calculations.

Uses decimal skills as required in managing personal finances:

- completes banking forms/transactions (e.g., deposits/withdrawals, writes a cheque)
- determines interest on money saved or borrowed
- monitors gross earnings and net pay
- reconciles bank statements
- prepares a budget.

Related Applications Across the Curriculum

English

Recognizes the use of decimals in the Dewey decimal system.

Uses comprehension skills in reading/understanding problems that involve whole numbers and decimals.

Science

Reads and interprets decimal calibrations on meters, gauges and scales.

Uses decimals when interpreting/analyzing data obtained from observations/experiments.

Occupational Courses

Uses paper-and-pencil computation, mental arithmetic, estimation and the calculator to solve problems that involve:

- reading and interpreting numbers represented on meters, gauges and scales
- reading and interpreting numbers in the context of purchasing/selling/inventory control and in following formulas/recipes
- handling of billings, invoices and cash/credit transactions (e.g., prepares work orders, prepares bills for service, prepares bills of sale, makes change, balances cash receipts)
- calculating multiple costs (e.g., units × price = total cost). Checks calculations through estimation
- calculating unit cost (e.g., total cost ÷ number of units = unit cost). Checks calculations through estimation
- calculating wages earned based on hourly, daily, weekly, monthly or annual schedules
- making financial plans/preparing budgets.

Suggested Strategies/Activities

Decimal skills should be developed and reinforced within the context of their application to themes suggested in the *Teacher Resource Manual*. For example, place value need only be studied to the hundredth's place when dealing with money in "Budgeting and Banking". If examining the use of a micrometre in "Math in the Workplace", it would be necessary to study place value to the thousandth's place.

While students should understand process and be able to use appropriate algorithms when the number of digits is small, calculators should be used on a regular basis in applications and problem solving that involve large numbers or tedious calculations. Assist students to interpret the multi-digit decimals that may be displayed as the result of certain computations performed on the calculator. Estimation strategies must be developed and used in checking the reasonableness of answers obtained on the calculator.

Model techniques for mental arithmetic that include the use of compensation strategies and computing from left to right (see *Teacher Resource Manual*: Computational Facility and Estimation). Simulate consumer situations where students are required to use these techniques in mentally computing:

- the total cost of several items purchased
- change due, given amount offered in tender
- unit price/multiple price
- simple interest on small amounts of money
- their gross earnings.

Evaluation techniques should focus attention on student ability to use all methods of computation in applications and problem solving (see *Teacher Resource Manual*: Assessment/Evaluation). While paper-and-pencil tests may be an effective way of evaluating some skills, assessment should also include:

- informal "demonstration" of understanding through discussion/project work
- observation of attitudes and performance
- participation in small group activities
- personal interviews.

LEARNING OBJECTIVES	Related Life Skills

FRACTIONS

DEMONSTRATES ABILITY TO USE BASIC FRACTION SKILLS IN PRACTICAL PROBLEM-SOLVING SITUATIONS (WITH EMPHASIS ON FRACTIONS HAVING DENOMINATORS OF 2, 3, 4, 5, 8, 10 AND 16).

- Illustrates a fraction as part of a whole, part of a set, or a point on a number line.
- Describes proper/improper fractions and mixed numbers through the use of objects, pictures and diagrams.
- Using a number line, illustrates the relationship between whole numbers, decimals and fractions.
- Compares and orders fractions in applications.
- Identifies and determines equivalent fractions (including proper and improper fractions, mixed numbers).
- Identifies and expresses fractions in basic form.
- Converts proper/improper/mixed fractions to decimals using a calculator.
- Recalls decimal equivalents for one-half, thirds, quarters, fifths and tenths.
- Determines common denominators for fractions having denominators of 2, 3, 4, 5, 8, 10 and 16.
- Adds/subtracts fractions and mixed numbers having line/unlike denominators that are within the parameters stated above.
- Multiplies/divides proper fractions and mixed numbers by whole numbers.
- Calculates a fractional part of an amount in applications.

Reads and interprets fractions represented on scales and gauges:

- inch ruler
- kitchen/workshop tools that measure capacity and mass
- fuel gauge.

Uses the inch unit in taking measurements for various construction/maintenance/repair projects in the home. Measures figures and objects to the nearest half, quarter, eighth and sixteenth of an inch.

Interprets fractional measurements provided on diagrams and blueprints. Finds missing dimensions by adding/subtracting fractional parts of an inch.

Follows recipes and directions that involve the use of fractional units of measure:

- two and three-eighth cups
- one and one-half teaspoons
- one-third of a package
- one-half of a block
- one-quarter of a turn.

Uses fraction skills in order to:

- describe a quantity or amount (e.g., two-thirds of a cup)
- make comparisons (e.g., two-year olds are approximately $\frac{1}{2}$ of their adult height)
- interpret/use Imperial measure
- determine discount/sale price (e.g., one-third off)
- calculate gross pay (e.g., $37\frac{1}{2}$ hours at $4.50/hour)
- calculate interest earned for fractional periods of time (e.g., $1000 invested at 10% for $1\frac{1}{4}$ years).

Adjusts recipes/patterns involving fractional units of measure:

- purchases enough material/accessories to make two shirts
- decreases a recipe for 8 servings to a recipe for 6 servings.

Related Applications Across the Curriculum

Science

Reads/interprets fractional calibrations on measurement tools.

Uses fractions when:

- describing personal health and lifestyle factors (e.g., pulse rate, nutritional requirements)
- determining concentration of mixtures/ solutions
- analyzing the mechanical advantage/ efficiency of various labour-saving technologies.

Occupational Courses

Uses fraction concepts and computation in solving problems that involve:

- reading/interpreting calibrations on metric/Imperial measuring instruments
- modifying quantities of supplies/ ingredients to suit variable needs (e.g., increasing/decreasing recipe ingredients, chemical mixes)
- reading charts to determine equivalent fraction-decimal sizes for mechanical fasteners and tools
- recognizing the size of various commonly used fasteners and tools (e.g., drill bits, wrenches, screws, nuts and bolts)
- reading and interpreting whole number, decimal and fractional measurements used in diagrams, blueprints, recipes and formulas.

Suggested Strategies/Activities

The cognitive demands of concepts related to fractions make it necessary that instruction include the extensive use of manipulative and visual materials.

Structure activities through a "math lab" approach, giving each student the opportunity to manipulate materials and see tangible outcomes. De-emphasize activities that yield singular results through the application of computational rules not yet understood by students. Select activities that encourage students to:

- work with real models/pictures/diagrams
- experiment/explore
- ask questions
- verbalize ideas and discoveries
- make associations/draw conclusions.

Keep arithmetical computation simple, working only with fractions commonly encountered in real life situations. Activities might be limited to work with the fractions included in the following families:

- 1/2's, 1/4th's, 1/8th's, 1/16th's
- 1/2's, 1/3rd's, 1/6th's
- 1/2's, 1/5th's, 1/10th's.

Demonstrate operations with fractions through concrete manipulation and diagramatic representation. Translation to numbers and symbols should occur only after the processes have been understood. By asking appropriate chains of questions (see "Instructional Mediation"), students can be encouraged to discover relationships and generalize the results of their investigations.

The Imperial ruler offers many opportunities for students to reinforce/extend their understanding of fractions in a practical context. Measuring/drawing figures and objects to the nearest 1/2, 1/4, 1/8 and 1/16 of an inch will enhance student ability to recognize order and equivalence among fractions. Operations can also be demonstrated on the ruler.

Strategies that support concept development through a "math lab" approach are provided in the *Teacher Resource Manual*: Situational and Concrete Approaches.

LEARNING OBJECTIVES	Related Life Skills

INTEGERS

DEMONSTRATES ABILITY TO USE INTEGER SKILLS WITHIN THE CONTEXT OF PRACTICAL SITUATIONS.

- Identifies situations that require the use of integers.
- Uses vocabulary related to integers (i.e., positive, negative, plus, minus, above, below, gain, loss).
- Places integers on the number line.
- Compares and orders integers in practical applications.
- Adds pairs/groups of one- and two-digit integers.

Interprets temperatures both above and below zero. Determines temperature change.

Recognizes application of integers in score-keeping for games and sports:

- golf scores above and below par
- goals for and against in a hockey game
- yards gained and lost in football
- points gained and lost in a card game.

Uses integers in maintaining a bank account balance. Understands the concepts of an "overdraft" and "NSF cheque".

Recognizes the use of integers in describing:

- altitudes above and below sea level
- changes in the stock market
- time zone changes.

Related Applications Across the Curriculum

Science

Applies integer skills in monitoring temperature and calculating temperature change.

Social Studies

Relates the use of integers to:

- altitudes above and below sea level
- income and deductions.

Occupational Courses

Applies knowledge of integer concepts to situations that involve:

- reading and interpreting temperatures above and below zero
- calculating temperature change
- calculating profit and loss in production/service oriented activities (e.g., merchandising lab, hair and beauty services, automotive services)
- interpreting tolerances on various tools/ gauges.

Suggested Strategies/Activities

Integer concepts and skills should be developed within the context of their application to themes suggested in the *Teacher Resource Manual*. Encourage students to explore a variety of problem situations that require the use of integers:

- changes in the stock market
- keeping score in game situations
- debits and credits.

The abstract nature of integer operations makes it necessary that instruction involve the extensive use of manipulative and visual materials. Structure activities using a "math lab" approach, and encourage students to:

- work with real models/pictures/diagrams
- experiment/explore
- verbalize ideas and discoveries
- make associations/draw conclusions.

Addition of integers should be demonstrated through concrete manipulation and diagramatic representation. Summarize the results of these investigations through the use of numbers and symbols. Encourage students to develop rules for adding integers after the processes have been understood.

Activities that are useful in developing an understanding of integer operations through the concrete mode are provided in the *Teacher Resource Manual*: Situational and Concrete Approaches.

MATHEMATICS 16

RATIO, PROPORTION AND PERCENT

Students will be expected to demonstrate an understanding of ratio, proportion and percent, and use related concepts and skills in evaluating media claims and making informed decisions on buying, selling or investing. Instruction at the intuitive and concrete level will assist students to internalize concepts and apply related skills in real life situations.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- a ratio is a comparison of the relative sizes of two quantities
- a rate is a ratio between quantities that have different units
- proportions are statements about equivalent ratios
- percent is a ratio in which a quantity is compared to 100.

SKILLS

The skills relating to RATIO, PROPORTION AND PERCENT are outlined in the Learning Objectives listed in Column One on the following page.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward the application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- appreciate the usefulness of ratio, proportion and percent skills in solving consumer problems
- appreciate the importance of computational facility and critical thinking skills in evaluating information and making informed decisions in daily life and work-related situations.

COURSE: MATHEMATICS 16
RATIO, PROPORTION AND PERCENT

LEARNING OBJECTIVES	Related Life Skills

DEMONSTRATES ABILITY TO USE RATIO, PROPORTION AND PERCENT SKILLS IN EVERYDAY APPLICATIONS AND PROBLEM-SOLVING SITUATIONS.

- Compares two quantities in the same unit by writing ratios in the form a/b, a:b and a to b.
- Generates equivalent ratios.
- Calculates rates/unit rates by writing ratios that involve numbers with different units (e.g., 90 km/2h, 3 items for $1.00).
- Identifies proportions as statements about equivalent ratios.
- Writes proportions that describe practical problem situations.
- Calculates the unknown value in a proportion.
- Expresses whole number percents as ratios/decimals.
- Expresses ratios as decimals and percents (i.e., $\frac{a}{b} = \frac{?}{100}$ where b = 2, 4, 5, 10, 20, 25 or 50).
- Recalls fraction, decimal and percent equivalents for one-half, quarters and tenths.
- Expresses one-, two- and three-place decimals as percents.
- Determines other fraction, decimal and percent equivalents through the use of the calculator.
- Calculates/estimates a percent of a number.

Recognizes travel rate as a comparison of distance travelled and time required for the journey.

Applies ratio skills to travel situations, recognizing the relationship between distance travelled, travel rate and travel time. Uses equivalent ratios to solve related problems.

Uses equivalent ratios in order to:

- determine unit cost and multiple cost of consumer items
- determine correct amount of solute and solvent for solutions
- interpret/construct scale drawings and models (e.g., maps, blueprints).

Interprets the meaning of percent in applications that relate to:

- discounts/sale prices
- price increase
- school marks
- interest on loans/charge accounts/ savings accounts
- tax rates
- commissions.

Recognizes fractional and percent equivalents for discounts advertised on consumer items (e.g., a 25% sale means one-quarter off).

Estimates money saved and the sale price on an article when given the percent discount rate.

Calculates simple interest. Recognizes the effect of interest rate on the payback amount of the loan (or on interest accumulated from an investment).

Uses estimation in determining:

- interest from a bank account
- gross pay increase given a wage increase of 3%.

Related Applications Across the Curriculum

Science

Uses ratio/proportion skills in monitoring personal health (e.g., heart-beat recovery rates, nutritional content of foods).

Uses ratio/proportion/percent in recognizing the mechanical advantage and efficiency of various machines and technologies.

Uses knowledge of ratio and percent in interpreting data related to pollution factors.

Uses ratio in preparing chemical solutions and mixtures. Modifies quantities according to need.

Social Studies

Uses skills in ratio/proportion/percent when:

- interpreting the scale provided on a map
- interpreting/calculating data related to topics that are investigated (e.g., employment trends, voting results, traffic violations, credit card use)
- comparing warranties on consumer products.

Occupational Courses

Uses ratio and proportion skills when:

- working with mixtures and solutions
- working with scale drawings or models
- constructing/applying gear and pulley systems
- increasing/decreasing recipes.

Calculates percent when:

- determining mark-up and selling price
- determining discount and sale price
- measuring profit or loss.

Calculates profit and loss as a percent. Recognizes the relationship between percent profit and entrepreneurial success.

Suggested Strategies/Activities

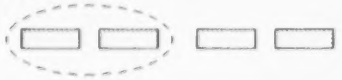
It is often assumed that high school students understand the concepts of ratio, proportion and percent because they have been exposed to these ideas throughout their school experience. This may not be the case, and care must be taken to develop these concepts through meaningful activities. Diagnose areas of student strength/weakness, and plan remedial activities that provide concrete and visual experiences for students.

Keep number relationships simple, and emphasize relevant applications rather than tedious computation. Develop concepts through learning activities that involve:

- discount/mark-up
- scale drawings
- pay-cheque deductions
- interest
- unit pricing/multiple pricing
- gear ratios
- wage rates/commissions.

Students should recognize that numbers have different equivalent forms (e.g., fraction, decimal and percent). Emphasize the use of mental arithmetic in determining fraction, decimal and percent equivalents that are frequently used.

Encourage students to develop strategies for solving proportions and finding percent by investigating a variety of concrete representations.
e.g., 50% of $4.00



The "cross-product" method of solving proportions should be introduced after students have experience in solving proportions using the common multiple/common factor methods.

Relate calculations that are performed to real life situations. Collect newspaper articles or sale flyers that use ratio and percent (e.g., 1/3 discount, 25% increase, 10% interest) and simulate situations that require the application of related concepts and skills.

Once processes are understood, students should use the calculator in applications and problem solving. Provide instruction on using the percent key, and model the sequence of entries that are made in performing routine calculations with ratio and percent. Emphasize the use of estimation in checking the results of computation.

Maintain/extend mental arithmetic and estimation skills by providing frequent opportunity for oral work:

- 50% of $12.50
- a $20.00 shirt discounted by 10%
- 15 out of 20 = __%.

MATHEMATICS 16

ALGEBRA

Students will be expected to demonstrate an understanding of algebraic thought and process by generalizing arithmetical patterns and relationships that are present in concrete situations, and by writing expression/equations that describe the patterns and relationships they discover. Students will use substitution and equation-solving strategies to solve real life and work-related problems that involve familiar number patterns and relationships.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- arithmetical patterns and relationships are present in a variety of practical everyday situations
- algebraic symbols can be used to write expressions/formulas/linear equations that describe familiar arithmetical patterns and relationships
- substitution and equation-solving strategies are useful in solving practical problems, and involve application of number properties and operations.

SKILLS

The skills relating to **ALGEBRA** are outlined in the Learning Objectives listed in Column One on the following page.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward the application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- appreciate the usefulness of algebra in describing arithmetical patterns and relationships that are present in practical situations, as well as in solving daily life and work-related problems.

COURSE: MATHEMATICS 16
ALGEBRA

LEARNING OBJECTIVES	Related Life Skills

DEMONSTRATES ABILITY TO USE ALGEBRAIC SKILLS WITHIN THE CONTEXT OF EVERYDAY AND WORK-RELATED SITUATIONS.

- Distinguishes between the use of variables and constants in practical situations.
- Uses variables to write mathematical expressions/equations that describe familiar situations.
- Evaluates expressions for given whole number values of the variable.
- Uses estimation, guess/check, concrete manipulatives and formal strategies to find whole number solutions for simple equations (e.g., $x \pm a = b$, $ax = b$, $x/a = b/c$, $ax \pm b = c$).
- Applies equation-solving skills to practical problem situations.
- Interprets formulas as word statements.
- Uses formulas and substitution to solve problems in practical situations (e.g., if $P = 2l + 2w$, find P when $l = 10$ and $w = 8$).

Recognizes number patterns and relationships that are present in practical situations:

- home decorating/maintenance/repair
- personal finance/management
- travel/sport/recreation.

Interprets simple linear equations/formulas that describe number patterns and relationships present in familiar situations. Relates equations/formulas to word statements about the situation:

- $A = lw$
- $V = lwh$
- $I = prt$
- $A = p + I$
- $d = rt$.

Uses equations/formulas to solve routine problems that involve previously established number patterns and relationships. Problem situations may involve:

- gross pay, deductions and net pay
- calculation of interest
- unit price and multiple price
- discount and sale price
- mark-up and selling price
- scale drawings
- perimeter/area/volume
- conversion among units of measure
- adjustments to recipes/directions
- travel (i.e., distance, rate, time).

Related Applications Across the Curriculum

Science

Writes equations that describe numerical relationships discovered through scientific investigation.

Performs substitution into formulas as required in order to solve problems.

Social Studies

Uses algebraic notation to describe:

- population/employment trends
- inflation rates
- income and deductions.

Occupational Courses

Uses formulas/equations to solve problems that involve:

- food requirements for a variable number of people
- stock requirements based on past sales
- conversions among units of measure
- relationships among travel distance, rate of travel and travel time
- adjusting recipes/mixtures according to need
- profit/loss in entrepreneurial activities.

Suggested Strategies/Activities

Algebraic thought and process should be developed throughout the mathematics program within the context of themes provided in the *Teacher Resource Manual*. Activities should foster an understanding of:

- number patterns/relationships present in real life situations
- strategies for evaluating expressions and formulas that describe practical situations
- both informal and formal strategies for solving simple equations.

Encourage students to generalize numerical patterns and relationships by constructing tables. Look for patterns, and write expressions that describe the patterns.

Provide opportunities for students to evaluate expressions/formulas that describe problem situations by using a variety of strategies:

- mental arithmetic
- formal substitution
- calculator
- computer programs.

Relate "guess and check" strategies for solving equations to finding numbers that balance the left and right sides of the equation. Emphasize the need to organize work and document processes used through formal substitution procedures.

Example: Solve $2n + 3 = 15$

Try $n = 5$ $2(5) + 3 = 13$ (too small)
Try $n = 8$ $2(8) + 3 = 19$ (too big)
Try $n = 6$ $2(6) + 3 = 15$ (just right)

Encourage the use of the calculator (and the automatic constant function) as an aid to the process of finding solutions by guess/check.

Assist students to develop "formal" strategies for solving equations by comparing the process to:

- maintaining balance on a scale (e.g., a number added to one side of the equation must also be added to the other side
- packing and unpacking boxes (e.g., reversing operations and the order in which they are performed).

Equation-solving strategies of this nature are described in further detail in the *Teacher Resource Manual*: Problem Solving.

MATHEMATICS 16

GEOMETRY AND MEASUREMENT

Students will be expected to demonstrate an understanding of the attributes/properties of familiar one-, two- and three-dimensional geometric figures, and to apply related concepts and skills in real life and work-related situations. Students will also be expected to estimate and measure length, area, volume, capacity, mass, time and angles, selecting units and tools appropriate to the situation.

It is the policy of Alberta Education that SI units be the principal system of measurement in the curriculum of the schools in the province. The study of specific Imperial units should be related only to those that are relevant to student needs (as indicated by the demands of the workplace/community partnership sites) and should be kept to a minimum.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- geometry is a visual approach to organizing and interpreting our perceptions of the environment and physical world
- there are basic concepts, patterns and relationships associated with one-, two- and three-dimensional geometric figures
- the selection of units and tools of measurement must always be based upon the physical attributes of the object being measured
- estimation and measurement are iterative and comparative in nature
- a mental frame of reference is useful in establishing the size of standard units of measure, relative to each other and to real objects
- geometry and measurement have application in everyday activities that involve repair, design and/or construction.

SKILLS

The skills relating to **GEOMETRY AND MEASUREMENT** are outlined in the Learning Objectives listed in Column One on the pages that follow.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward the application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- appreciate how geometry and measurement provide us with methods for gathering and organizing information about the physical world
- appreciate the ways in which geometry and measurement affect our daily activities and contribute to the problem-solving process.

COURSE: MATHEMATICS 16
GEOMETRY AND MEASUREMENT

LEARNING OBJECTIVES	Related Life Skills

GEOMETRY

DEMONSTRATES ABILITY TO APPLY GEOMETRY SKILLS IN DAILY LIFE AND WORK-RELATED SITUATIONS.

- Identifies line relationships (i.e., horizontal, vertical, parallel, perpendicular and intersecting).
- Identifies/constructs basic two-dimensional geometric figures (i.e., rectangle, square, parallelogram, triangle).
- Classifies triangles and quadrilaterals by examining measures of sides and measures of angles.
- Identifies the relationship between the radius and diameter of a circle.
- Constructs circles, given either radius or diameter.
- Identifies similar and congruent geometric figures.
- Identifies/sketches/constructs models of basic three-dimensional shapes (i.e., rectangular solid, cube, cylinder, cone).
- Constructs geometric figures/designs using a variety of construction tools (i.e., straightedge, ruler, compass, protractor, computer).

Recognizes/appreciates geometric form found in:

- the natural environment
- architecture
- floor coverings/fabrics/wallpaper.

Interprets/uses line relationships when reading road maps and giving travel directions.

Recognizes geometric shapes and relationships that are used:

- on traffic signs
- as symbols on maps
- on clothing care labels
- on hazardous materials/warning labels.

Applies knowledge of geometric figures and relationships when interpreting/constructing:

- scale diagrams (e.g., floor plans, maps)
- graphs and charts
- patterns and designs.

Interprets three-dimensional objects that have been represented by a sketch:

- recognizes perspective
- relates front, side and top views.

Enlarges or reduces patterns/scale drawings required for a project through the use of:

- dot paper/grid paper
- measurement and ratio.

Uses geometric figures/relationships in designing various home and hobby project. Plans the project by constructing a scale

Related Applications Across the Curriculum	Suggested Strategies/Activities
English Uses appropriate skills when following instructions to construct designs or scale drawings: • recognizes the need for sequence • interprets verbal and written instructions • interprets diagrams, signs and symbols in order to follow instructions. Science Recognizes/appreciates geometric patterns occurring in the natural environment. Social Studies Applies concepts of one-, two- and three-dimensional geometry when: • identifying national, provincial and municipal voting jurisdictions • locating specific addresses on a rural/urban grid system • preparing/interpreting scale drawings/ maps. Occupational Courses Uses geometric concepts/skills when completing projects in: • construction/repair/maintenance • craft work/artwork/graphic arts • hair care/esthetology • horticulture. Uses appropriate geometric tools in preparing scale drawings and completing projects.	Geometry should emphasize the visual perception of pattern and form present in the student's personal environment. Knowledge of geometric figures/relationships can be developed by modelling various aspects of the physical world, and by providing frequent opportunities for work with concrete materials. Teaching strategies that are inductive and experimental will enable students to recognize patterns and develop an understanding of the more abstract concepts in geometry. Activities should place emphasis on observation, manipulation and construction. Concepts of one-dimensional space are usually well understood. Students find two-dimensional and three-dimensional concepts more difficult, and may require concrete support for successful experience in these areas. Manipulation and construction will assist students in recognizing the spatial relationships inherent in these figures. Provide opportunities for students to construct geometric patterns/designs/logos using a variety of tools and techniques. These activities reinforce the geometric concepts under investigation, develop facility in the use of construction tools, and provide meaningful application of skills in linear and angle measure. Encourage students to be creative. Display student work in colourful and attractive arrangements. Discuss your geometry and measurement unit with teachers of other programs. Identify projects for which students might create patterns or scale drawings. A variety of instructional activities that may assist students to develop skills in visual perception are provided in the *Teacher Resource Manual*: Situational and Concrete Approaches. Activities include the use of: • geoboards/dot paper/grid paper • tangrams/tessellations • LOGO computer programs • line design/model construction • puzzles/problems.

LEARNING OBJECTIVES	Related Life Skills

LINEAR MEASURE

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE LENGTH IN DAILY LIFE AND WORK-RELATED SITUATIONS.

- Identifies common SI units of length (i.e., mm, cm, m, km).
- Estimates/measures length, using units and tools appropriate to the situation.
- Converts among SI units of length as required in applications.
- Estimates/measures/calculates the perimeter of figures bounded by line segments, using units and tools appropriate to the situation.
- Identifies the value of π as the ratio of the circumference of a circle to its diameter.
- Develops/applies strategies for determining the circumference of a circle.

E • Identifies common Imperial units of length (e.g., inch, foot, yard, mile).

E • Estimates and measures length, using Imperial units and tools appropriate to the situation.

E • Compares cm to inch, m to yard and km to mile.

Reads/interprets scales on rulers and tape measures.

Uses estimation skills in:

- discriminating between objects of "greater" and "lesser" length
- approximating the length of familiar household and consumer items.

Takes body measurements, using tools and units appropriate to the task. Relates personal measurements to clothing size.

Uses skills of estimation and measurement in determining materials required for various decorating, construction and repair projects.

Converts units of measure as required in determining the cost of consumer materials sold by length (e.g., 75 cm of fabric at a cost of $15.98/m).

Develops a "frame of reference" for the kilometre, and relates familiar travel distances to the kilometre.

E - Elective Content

Related Applications Across the Curriculum	Suggested Strategies/Activities

Science

Uses linear measurement skills when:

- collecting data from scientific investigations
- constructing simple models/technologies in project work.

Social Studies

Uses units/tools of linear measure when:

- estimating/calculating distances on a map
- drawing a map to scale
- developing bar/line graphs that display information.

Occupational Courses

Reads scales on rulers, tape measures and other measuring devices when building a product or providing a service.

Recognizes and applies concepts of length/perimeter/circumference in projects related to:

- construction trades
- agriculture/horticulture
- window display projects
- hair care/esthetology.

Converts among units of length as required in practical situations.

Measurement must be understood as a process of comparison to some arbitrary unit. Students need to recognize the repetition of identical units that occurs in measurement, and the need to combine/subdivide selected units into larger/smaller units when describing particular attributes. Encourage students to develop a "feel" for standard units through the use of referents and visual imagery (e.g., body referents, objects in the classroom/home). Provide opportunities for students to develop skill in making reasonable estimates of measure before engaging in activities that involve actual measurement and precision.

Consider the following guidelines in planning activities in estimation and measurement:

- develop skill in estimation by having students make an estimate and then check their estimate through direct measurement
- although some experience in direct measurement may immediately follow the introduction of a particular unit, only a few measurements of this nature should be taken before students are asked to make estimates prior to performing direct measurement
- encourage good estimates, but do not provide penalties for inaccurate ones
- the ability to estimate is based on previous experience and must be practised on a regular basis throughout the program.

Review standard units of length through the use of referents and visual imagery:

- mm (width of a dime)
- cm (width of little finger)
- m (width of a table)
- km (distance between two familiar points).

Relate activities in estimation and measurement to situations encountered in the real world:

- body measurements
- size of familiar objects/pieces of furniture
- dimensions of windows/doors/rooms
- distances in the community.

Business and industry have reached varying states in their conversion from Imperial units to metric units. Some occupations/life skills still require the use of Imperial units. Community needs will determine the extent to which elective content dealing with Imperial measure will be developed. If developed, students should compare only, and not convert corresponding units from the Imperial system to the metric system:

- "Is three metres of fabric more than three yards of fabric?"
- "Is my waistline measurement 30 inches or 30 cm?"

COURSE: MATHEMATICS 16
GEOMETRY AND MEASUREMENT

LEARNING OBJECTIVES	Related Life Skills
AREA MEASURE	
DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE AREA IN DAILY LIFE AND WORK-RELATED SITUATIONS.	Uses estimation/measurement/calculation in determining surface area and quantity of materials required for various maintenance projects around the home:
• Identifies common SI units of area (i.e., cm^2, m^2) and their application in practical situations.	• painting • carpeting • wallpapering • tiling.
• Approximates the area of two-dimensional geometric figures using a square grid.	Interprets product labels that relate "rate of application" to area:
• Estimates/calculates area of the rectangle, square, triangle and circle, using units, tools and strategies appropriate to the situation.	• lawn seed and fertilizer • paint.
	Develops a strategy for determining the amount of material required to cover a rectangular solid or cylinder:
	• gift wrap for a box • paint for a cylindrical storage tank.
VOLUME MEASURE	
DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE VOLUME IN DAILY LIFE AND WORK-RELATED SITUATIONS.	Recognizes the use of volume measure in describing:
• Identifies common SI units of volume (i.e., cm^3, m^3) and their application in practical situations.	• engine size • size of storage containers • refrigerator/freezer size • utilities consumed (e.g., natural gas, water).
• Approximates the volume of rectangular solids using cubes.	Determines the volume of concrete required for simple construction projects at home.
• Estimates/calculates the volume of rectangular solids and cubes, using units, tools and strategies appropriate to the situation.	Estimates/calculates the volume of containers frequently used around the home:
	• storage boxes • refrigerator/freezer.
	Interprets product labels that relate "rate of application" to volume measure:
	• fertilizers/growth supplements • medications.

Related Applications Across the Curriculum

Science

Acknowledges the relationship between millilitres and cubic centimetres.

Uses area/volume measure as required in gathering/interpreting data for scientific investigations.

Social Studies

Compares the areas of various sections within the community (e.g., industrial, residential, commercial, parkland).

Uses area concepts when comparing the population density of municipal, provincial and national jurisdictions.

Occupational Courses

Recognizes appropriate units and strategies for calculating area and volume.

Calculates area and volume in order to determine materials needed to produce a product or provide a service:

- construction projects
- agriculture/horticulture projects
- maintenance tasks
- hair care and esthetology services
- crafts, arts and graphic projects.

Suggested Strategies/Activities

Students may require concrete support in developing an understanding of the two-dimensional concept of area and the three-dimensional concept of volume. Activities should include the use of manipulative and visual materials:

- tiles/grid paper
- cubes/models.

Encourage students to develop "mind pictures" for the square centimetre, square metre, cubic centimetre and cubic metre. Have students construct these units.

Area and volume formulas should be developed as a consequence of patterns/relationships that have been discovered through manipulation and construction. Encourage students to deduce their own strategies for determining area and volume that are based on the results of investigation.

Distinguish between area (surface covered) and volume (space occupied). Illustrate the difference by referring to practical situations and identifying the concept involved:

- "How much carpet do we need?"
- "How much concrete do we need?"

Model the use of pictures/diagrams as a strategy for problem solving.

Appropriate instructional strategies for developing area and volume concepts are provided in the *Teacher Resource Manual*: Situational and Concrete Approaches. Relevant applications of area and volume are provided throughout themes developed in this manual.

Conferencing with occupational teachers will assist in establishing a meaningful context for lessons in measurement. Identify the measurement skills that are used by students in their occupational courses. Provide opportunities for students to develop and apply these skills in mathematics class.

LEARNING OBJECTIVES	Related Life Skills

CAPACITY MEASURE

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE CAPACITY IN DAILY LIFE AND WORK-RELATED SITUATIONS.

- Identifies common SI units of capacity (i.e., mL, L) and their application in practical situations.
- Estimates and measures capacity, using metric units and tools appropriate to the situation.
- Converts between mL and L.

E • Identifies common Imperial units of capacity (e.g., teaspoon, tablespoon, cup, quart, gallon).

E • Estimates and measures capacity, using Imperial units and tools appropriate to the situation.

E • Compares mL and L to cups, also L to quarts/gallons.

MASS MEASURE

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE MASS IN DAILY LIFE AND WORK-RELATED SITUATIONS.

- Identifies common SI units of mass (i.e., g, kg, t) and their application in practical situations.
- Estimates and measures mass, using metric units and tools appropriate to the situation.
- Converts between g and kg, and between kg and t.

E • Identifies common Imperial units of mass (e.g., ounce, pound).

E • Estimates and measures mass, using Imperial units and tools appropriate to the situation.

E • Compares kg to pound.

Reads and interprets scales on familiar measuring tools:

- kitchen/supermarket weight scale
- capacity measures used in the kitchen/workshop.

Uses estimation skills when:

- discriminating between objects of "greater" and "lesser" capacity/mass
- approximating the capacity/mass of familiar household and consumer items.

Selects items for purchase that are sold by capacity/mass. Estimates and measures consumer items in relation to these attributes, using units and tools appropriate to the situation.

Interprets product labels that relate "rate of application" to capacity/mass.

Converts units of measure as required when:

- adjusting recipes/directions that involve measure of capacity/mass
- determining unit price and "best buy" for consumer items sold by capacity/mass
- determining the cost of materials sold by capacity/mass (e.g., 250 g of meat at $4.98 per kg).

Uses capacity measure when:

- cooking
- taking medications.

Monitors personal weight, using appropriate units and graphing skills.

E - Elective Content

Related Applications Across the Curriculum	Suggested Strategies/Activities
Science Estimates/measures capacity and mass in a variety of inquiry-related activities: • measures chemical substances by capacity and mass • prepares mixtures/solutions using appropriate measures of capacity and mass • measures force/mass when investigating simple technologies. Occupational Courses Uses appropriate units and tools for measuring capacity and mass when: • following recipes • preparing mixtures/solutions • purchasing goods by capacity or weight. Converts between units as required.	Provide opportunities for the review of standard units of capacity/mass through the use of referents and visual imagery: • mL (amount of liquid in an eyedropper) • L (amount of milk in a carton) • g (mass of a raisin) • kg (mass of a pair of shoes). Strategies for making estimates of capacity/mass may include: • comparing the whole object to be estimated with a familiar object of known size • using a referent of known size and mentally or physically marking-off/filling the object to be estimated. Be sure to accept a range of estimates, and encourage students to share the strategies they use in making their estimates. Sequence activities so that students can improve the accuracy of the estimates they are making. Record a "+" each time an estimate is larger than the corresponding measurement, and a "–" each time the estimate is smaller. Students who consistently have pluses (minuses) will then realize that they need to adjust their estimates by making them smaller (larger). Provide frequent opportunities for students to estimate/measure through activities that simulate real life situations such as: • determining the mass of familiar bulk/packaged supermarket and hardware items • determining the capacity of kitchen/workshop containers of varying size and shape • following recipes/directions that require the use of capacity/mass measures. Community needs will determine the extent to which elective content dealing with Imperial measure will be developed. If developed, students should compare only and not convert corresponding units from the Imperial system to the metric system: • Does a one-gallon container hold more than a one-litre container? • Is a 5 kg bag of apples larger than a 5 pound bag of apples?

LEARNING OBJECTIVES	Related Life Skills

TIME MEASURE

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE TIME IN DAILY LIFE AND WORK-RELATED SITUATIONS.

- Uses a calendar, recognizing the relationship between days, weeks, months and years.
- Uses National Standards for numeric dating.
- Estimates/measures/records time on the 12-hour and 24-hour clock, using traditional and digital time pieces.
- Converts between hours and minutes, and between minutes and seconds.
- Determines the interval between a pair of times.
- Determines the finishing time, given the starting time and duration.

Uses calendar and numeric-dating skills in a variety of everyday situations:

- recognizes due dates on bills/library books
- dates letters/cheques/application forms appropriately
- identifies current and post-dated cheques.

Reads and interprets 24-hour time stated on:

- train/bus/airplane schedules
- traffic/parking signs.

Records start and finish times for tasks undertaken. Accurately determines/measures time intervals:

- determines cooking/baking times
- sets time-bake oven/video recorder
- determines hours and minutes worked.

ANGLE

DEMONSTRATES ABILITY TO USE APPROPRIATE UNITS AND TOOLS TO ESTIMATE AND MEASURE ANGLES IN DAILY LIFE AND WORK-RELATED SITUATIONS.

- Identifies an angle and the degree as a unit of measure.
- Identifies/estimates angles of 30°, 45°, 60°, 90°, 180° and 360°.
- Uses a protractor to measure and draw angles from 0° to 180°.

Recognizes angle relationships used in practical everyday situations. Appreciates their use in construction and design.

Applies skills of angle measure when:

- giving or following directions on a map
- reading a compass
- interpreting/constructing scale drawings
- constructing patterns/designs.

Related Applications Across the Curriculum	Suggested Strategies/Activities
Occupational Courses Maintains personal record of attendance and punctuality for classroom activities and field experiences: • time-in/time-out • hours worked • time of telephone contacts • record of appointments. Measures intervals of time required for various processes/procedures: • cooking/baking time • drying time • curing time.	Maintain skills in time measure by incorporating the use of the calendar, stopwatch, digital clock, and traditional 12-hour and 24-hour clocks into daily activities. Develop ability to estimate time by: • comparing time intervals to the duration of a familiar event (e.g., length of a song) • providing opportunities for students to estimate both active and passive periods of time. Projects that involve the use of time measure are provided in themes developed in the *Teacher Resource Manual*. These projects require students to interpret and use: • personal time schedules/time cards • travel/telecast schedules.
Applies skills of estimation and measurement when working with angles in a variety of occupational areas: • drafting/graphic arts • haircutting • window display • crafts and arts • construction fields.	Identify examples of angles found in the environment. Relate angles of 45°, 90°, 180° and 360° to physical models that are familiar to the student. Provide opportunities for students to estimate angle measure, and to check estimation through actual angle measure with a protractor. Instruction on the use of the protractor should include discussion of: • the purpose of the inside and outside scale • strategies to follow in measuring angles greater than 180°. Develop skill in angle measure by constructing geometric figures, patterns/designs and scale drawings.

MATHEMATICS 16

DATA INTERPRETATION AND DISPLAY

Students will be expected to demonstrate an understanding that data collected from a sample must be organized, displayed and analyzed in order that valid inferences can be drawn. Students will develop knowledge of the use of statistical data in a variety of everyday situations, and how statistical data affects their daily activities.

CONCEPTS

Students will be expected to demonstrate an understanding that:

- a variety of procedures can be used to collect, organize and display numerical data
- data collected from a sample must be organized, displayed and analyzed in order that valid inferences can be drawn
- statistical measures are useful in summarizing large quantities of numerical data and in communicating ideas
- relevant statistical data may assist an individual to solve practical problems and make informed decisions in personal and work-related situations.

SKILLS

The skills relating to **DATA INTERPRETATION AND DISPLAY** are outlined on the Learning Objectives listed in Column One on the following page.

ATTITUDES

Students will be encouraged to:

- display a positive attitude toward application of concepts and skills in mathematics by
  - showing interest and curiosity through willingness to ask questions, share observations and ideas, and seek answers
  - solving mathematical problems and completing assignments independently and in cooperation with others
- appreciate the need to analyze and evaluate statistical data provided through the media
- appreciate how statistical data may assist the decision-making process in consumer and work-related situations.

LEARNING OBJECTIVES

Related Life Skills

COLLECTS, ORGANIZES, DISPLAYS AND ANALYZES NUMERICAL DATA IN ORDER TO DRAW VALID INFERENCES AND MAKE INFORMED DECISIONS IN EVERYDAY SITUATIONS.

- Distinguishes between a survey and census, also between a sample and population, identifying potential biases that may occur in surveys and samples.
- Interprets and analyzes data presented in tables/charts and bar/line/picture/circle graphs.
- Displays data in the form of tables/charts and bar/line/picture/circle graphs.
- Identifies the characteristics of a misleading graphical display.
- Determines/calculates appropriate measures of central tendency in practical situations (i.e., mean, median, mode, range).
- Conducts a survey or poll, using appropriate methods of gathering, organizing, presenting and analyzing data:
  - selects a suitable sample from a given population
  - collects and organizes data using tally sheets/ frequency tables
  - displays data in the form of bar/line/picture/ circle graphs
  - makes inferences/generalizations about the population from which a sample was taken.

Reads/interprets/analyzes information provided in newspapers and magazines that has been displayed in table/chart/graph form.

Recognizes how statistics are used in everyday situations:

- weather reports (e.g., probability of precipitation)
- consumer reports (e.g., average price of an item)
- opinion polls (e.g., popularity of a television program or political figure)
- performance in sports (e.g., batting average)
- personal development (e.g., average height/weight charts)
- health risks (e.g., frequency of lung cancer in smokers)
- lottery outcomes (e.g., odds of winning the jackpot).

Uses appropriate measures of central tendency in describing:

- test marks/school achievement
- achievement in sports
- income over several weeks/months.

Recognizes potential biases that may be present in the results of surveys/polls used in advertising:

- 2/3 of the people interviewed prefer Brand X
- 8 out of 10 people felt the clothes washed by ___ came out whiter.

Related Applications Across the Curriculum

English

Reads/interprets information presented in tables/charts/graphs.

Interprets statistical data as required in research activities.

Science

Collects and records data through observation, interview and experimentation.

Organizes and displays data in table/chart/graph form.

Interprets information displayed in table/chart/graph form:

- personal health/fitness data
- temperature-volume and temperature-pressure relationships
- environmental/pollution factors.

Social Studies

Recognizes the frequent use of polls in the political arena. Analyzes the results of polls.

Gathers facts/opinions related to current political and law-related issues.

Occupational Courses

Reads and interprets graphs depicting trends in business and industry (e.g., job opportunities, production cycles, profits and losses).

Reads and interprets tables, charts and graphs in order to perform required tasks:

- engine performance graphs
- oven temperature conversion charts
- charts of body measurement/pattern size
- tap and die charts
- threading charts.

Reads and interprets symbols provided in table or chart form (e.g., hazardous and non-hazardous materials, road signs, fabric care signs).

Suggested Strategies/Activities

Plan activities throughout each theme that will enable students to recognize the use and misuse of statistics in real life situations. Select tables/charts/graphs from a variety of available sources that relate to the topics investigated. Identify related instances where statistics are used in the newspaper, or on television and radio.

Provide opportunities for students to experience the processes used in solving a statistical problem. Conduct surveys/polls within the community that relate to topics investigated within each theme. Assist students to develop appropriate strategies for:

- collecting their data
- organizing and displaying their data.

Once the data has been collected, organized, and displayed, ask questions that will encourage students to:

- identify potential biases in the data they have collected
- draw conclusions/make inferences that are based on the data they have collected
- identify characteristics of the display that may be misleading
- determine appropriate measures of central tendency.

Information provided in tables, charts and graphs is often useful in solving real life problems. Model the use of tables/charts/graphs as a problem-solving strategy, and assist students to:

- interpret problem information provided in table/chart/graph form
- manipulate/organize problem information into charts/graphs in order to make patterns and relationships more discernible.

Projects undertaken in other subject areas may provide additional opportunities for students to collect, organize, display and interpret numerical data within meaningful contexts.

END

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